CHP. 15

ELECTROMAGNETIC INDUCTION.

**In 1831, Michael Faraday in England and Joseph Henry in USA observed that:**

**Def:** "An emf is set up in a conductor when it is moved across a magnetic field which results in the electric current flowing through a circuit."

The emf produced in the conductor is called **induced emf**, and the current generated is called the **induced current**. This phenomenon is known as **electromagnetic induction**.

**PRODUCTION OF INDUCED EMF AND INDUCED CURRENT.**

Consider a straight piece of wire of length \( \ell \) placed in the magnetic field of a permanent magnet, as shown in Fig. 1. The wire is connected to a sensitive galvanometer. This forms a closed path or loop without any battery. In the beginning, when the loop is at rest in the magnetic field, no current is shown by the galvanometer.

If we move the loop from left to right, the length \( \ell \) of the wire is dragged across the magnetic field and a current flows through the loop. As we stop moving the loop, the current also stops. Now, if we move the loop in the opposite direction, the current also reverses its (P.T.O.)
direction. This is indicated by the deflection of the galvanometer in opposite direction.

**DEPENDENCE OF INDUCED CURRENT**

The induced current depends upon:

1. The speed with which the conductor moves
2. The resistance of the loop.

If we change the resistance of the loop, by inserting different resistors in the loop, and move it in the magnetic field with the same speed every time, we find that the product of induced current $I$ and the resistance $R$ of the loop remains constant i.e.; $IR = \text{const.}$

This constant is the induced emf. The induced emf leads to an induced current when the circuit is closed. The current can be increased by:

(a) Using a stronger magnetic field
(b) Moving the loop faster
(c) Replacing the loop by a coil of many turns.

If we perform the above experiment in the other way, i.e., instead of moving the loop across the magnetic field, we hold the loop stationary and move the magnet, then it is easily observed that the results are the same.

**CONCLUSION**

It is the relative motion of the loop and the magnet that causes the induced emf.

**CAUSE OF INDUCED emf.**

In fact, this relative motion changes the magnetic flux through the loop, therefore, we can say that an induced emf is produced in a loop if the magnetic flux through it changes. The greater the rate of change of flux, the larger is the induced emf.

(F.T.S.)
# DIFFERENT METHODS TO PRODUCE INDUCED EMF:

1. **By relative motion of bar magnet or coil.**

   A bar magnet and a coil of wire to which a galvanometer is connected as shown in Fig 2(a). When there is no relative motion between the magnet and the coil, the galvanometer indicates no current in the circuit. As soon as the bar magnet is moved towards the coil, a current appears in it as shown in Fig 2(b). In moving the magnet, the magnetic flux through the coil changes, and this changing flux produces the induced current in the coil. When the magnet moves away from the coil, a current is again induced but now in opposite direction. The current would also be induced if the magnets were held stationary and the coils were moved.

2. **Changing the area of the coil in a constant magnetic field.**

   There is another method in which the current is induced in a coil by changing the area of the coil in a constant magnetic field. Fig (3)(a) shows that no current

(P.T.W)
is induced in the coil of constant area that is placed in a constant magnetic field. However, when the coil is being distorted so as to reduce its area as shown in Fig 3(b), an induced emf and hence current appears. The current vanishes when the area is no longer changing. If the distorted coil is brought to its original circular shape thereby increasing the area, an oppositely directed current is induced which lasts as long as the area is changing.

3. **Coil of constant area is rotated in a constant magnetic field.**

   An induced current can also be generated when a coil of constant area is rotated in a constant magnetic field. Now the magnetic flux through the coil also changes. This is the basic principle used in electric generators.

   ![Diagram of a coil rotated in a constant magnetic field](image)

4. **Producing a change in magnetic flux in a nearby coil (Mutual induction).**

   An induced current in a coil can be produced by producing a change in magnetic flux in a nearby coil. Fig 5 shows two coils placed side by side. The coil ‘P’ is connected in series with a battery, a rheostat and a switch, while the other coil ‘S’ is connected to a galvanometer only. Since there is no battery in coil ‘S’,

   ![Diagram of mutual induction](image)
So current through it will always be zero. Now, if the switch of the coil P is suddenly closed, a momentary current is induced in coil S. This is indicated by the ammeter, which suddenly deflects and then returns to zero. No induced current exists in coil S as long as the current flows steadily in the coil P. An oppositely directed current is induced in the coil S' at the instant the switch of coil P is opened. Actually, the current in P grows from zero to max. value just after the switch is closed. The current comes down to zero when the switch is opened. Due to change in current, the magnetic flux associated with the coil P changes momentarily. This changing flux is also linked with the coil S. That causes the induced current in it. Current in coil S' can also be changed with the help of rheostat.

5. Changing magnetic flux in a coil by using an electromagnet.

It is also possible to link the changing magnetic flux with a coil by using an electromagnet instead of a permanent magnet. In this method, the coil is placed in the magnetic field of an electromagnet as shown in fig (a). Both the coil and the electromagnet are stationary. The magnetic flux through the coil is changed by changing the current of the electromagnet, thus producing the induced current in the coil.
# 15.2 MOational EMF:

Def. "The emf induced by the motion of a conductor across a magnetic field is called motional EMF."

EXPLANATION.

Consider a conducting rod of length $L$ placed on two parallel metal rails separated by a distance $L$. A galvanometer is connected between the ends $c$ and $d$ of the rails. This forms a complete conducting loop $abcd$ as shown in Fig. 1. A uniform magnetic field $B$ is applied directed into the paper. Initially, when the rod is stationary, the galvanometer indicates no current in the loop. If the rod is pulled to the right with constant velocity $v$, the galvanometer indicates a current flowing through the loop. Obviously, the current is induced due to the motion of the conducting rod across the magnetic field. The moving rod is acting as a source of emf.

$$E = V_b - V_a = \Delta V$$

when the rod moves, a charge $q$ within the rod also moves with the same velocity $v$ in the magnetic field $B$ and experience a force given by:

$$F = qvB \sin \theta$$

The magnitude of this force is:

$$F = qvb \sin \theta$$

Since angle $\theta$ between $v$ and $B$ is $90^\circ$, $\sin \theta = 1$.

(P.T.U.)
\[ F = q \mathbf{v} \mathbf{E} \]  
\[ (\because \sin \theta = 1) \]

Applying the right-hand rule, we see that \( \mathbf{F} \) is directed from \( \mathbf{a} \) to \( \mathbf{b} \) in the rod. This suggests that a uniform electric field \( \mathbf{E} \) is induced along the rod. Its magnitude is given by

\[ E = \frac{F}{q} \]  \hspace{1cm} (1)

Putting \( F = q \mathbf{v} \mathbf{E} \), we have

\[ E = \frac{q \mathbf{v} \mathbf{E}}{q} = \mathbf{v} \mathbf{E} \]  \hspace{1cm} (2)

The direction \( \mathbf{E} \) of electric intensity is that of force \( \mathbf{F} \), i.e., it is directed from \( \mathbf{a} \) to \( \mathbf{b} \).

As the electric intensity is given by the negative of the potential gradient, therefore,

\[ E = -\frac{\Delta V}{L} = -\frac{V}{L} \]  \hspace{1cm} (3)

Comparing eqs. (2) and (3), we have

\[ -\frac{V}{L} = \mathbf{v} \mathbf{E} \]

or \[ E = -\frac{V}{L} \mathbf{E} \]  \hspace{1cm} (4)

If the angle between \( \mathbf{V} \) and \( \mathbf{E} \) is \( \theta \), then the magnitude of motional emf is

\[ E = VBL \sin \theta \]  \hspace{1cm} (5)

The above eq. shows that when \( V = 0 \), \( E = 0 \), which means that no motional emf is produced in the stationary rod. It is also obvious that by increasing the speed of rod and using stronger field, emf can be increased.

Due to the induced emf, positive charges would flow along the path \( \mathbf{a} \mathbf{b} \mathbf{c} \mathbf{d} \mathbf{a} \), therefore, the induced current is anticlockwise in the diagram (1).

(P.T.O.)
EXAMPLE 15.1: A metal rod of length 25 cm is moving at a speed of 0.5 m/s in a direction perpendicular to a 0.25 T magnetic field. Find the emf produced in the rod.

**Data**
- Length of rod = L = 25 cm = 0.25 m
- Speed of rod = \( V = 0.5 \text{ m/s} \)
- Magnetic flux density = \( B = 0.25 \text{T} \)
- Induced emf = \( E = ? \)

**Sol.** Using the relation:

\[ E = VBL \]

\[ = 0.5 \times 0.25 \times 0.25 \]

\[ E = 3.13 \times 10^{-2} \text{ V} \]

**Point to Ponder**

1. 
2. 
3. 
4. 
5. 

A copper ring passes through a rectangular region where a constant magnetic field is directed into the page. What do you guess about the current in the ring at the positions 2, 3 and 4?

**Interesting Information**

This heater operates on the principle of electromagnetic induction. The water in the metal pot is boiling whereas that in the glass pot is not. Even the glass top of the heater is cool to touch. The coil just beneath the top carries ac current that produces changing magnetic flux. Flux linking with pots induce emf in them. Current is generated in the metal pot that heats up the water, but no current flows through the glass pan, why?
# 15.3 FARADAY'S LAW AND INDUCED EMF:

**Statement:** “The average emf induced in a conducting coil of 'N' loops is equal to the negative of the rate at which the magnetic flux through the coil is changing with time.”

**EXPLANATION:**

Consider the experiment shown in Fig. (1). Let the conducting rod 'L' move from position 1 to position 2 in a small interval of time \( \Delta t \). The distance travelled by the rod in time \( \Delta t \) is

\[ x_2 - x_1 = \Delta x \]

Since the rod is moving with constant velocity \( v \), therefore,

\[ v = \frac{\Delta x}{\Delta t} \]  \( \text{(1)} \)

Putting this value of \( v \) in motional emf eq., we have

\[ E = -vBL = -\frac{\Delta x \cdot BL}{\Delta t} \]  \( \text{(2)} \)

As the rod moves through the distance \( \Delta x \), the increase in the area of loop is given by

\[ \Delta A = \Delta x \cdot L \]  \( \text{(3)} \)

Therefore, flux through the loop becomes

\[ \Delta \Phi = B \cdot \Delta A = B \Delta x \cdot L \cos \theta = B \Delta x \cdot L \]  \( \text{(max. value)} \)

or

\[ \Delta \Phi = B \cdot \Delta x \cdot L \]  \( \text{(4)} \)

Putting this value \( \Delta x \cdot L \cdot B = \Delta \Phi \) in eq. (3), we have

\[ E = -\frac{\Delta \Phi}{\Delta t} \]  \( \text{(4)} \)

If there is a coil of 'N' loops instead of a single loop, then the induced emf will become \( N \) times. i.e.,

\[ (N \cdot \text{E.T.D.}) \]
This is the mathematical form of Faraday's law of electromagnetic induction. The negative sign indicates that the direction of the induced emf is such that it opposes the change in flux.

The induced emf is also defined as

\[ \mathbf{E} = -\frac{\Delta \Phi}{\Delta t} \tag{5} \]

**N.B.** Positive emf means that the direction of induced electric field in the loop will produce anticlockwise current, whereas negative emf means an induced field that produces clockwise current.

**EXAMPLE 15.2:** A loop of wire is placed in a uniform magnetic field that is perpendicular to the plane of the loop. The strength of the magnetic field is 0.6 T. The area of the loop begins shrinking at a constant rate of \( \frac{\Delta A}{\Delta t} = 0.8 \text{ m}^2 \text{s}^{-1} \). What is the magnitude of emf induced in the loop while it is shrinking?

**DATA.** Magnetic flux density = \( B = 0.6 \text{ T} \)

Rate of change of area = \( \frac{\Delta A}{\Delta t} = 0.8 \text{ m}^2 \text{s}^{-1} \)

Number of turns = \( N = 1 \)

Induced emf = \( \mathbf{E} = ? \)

**Sol.** We know that

\[ \text{Rate of change of flux} = \frac{\Delta \Phi}{\Delta t} = \frac{B \Delta A \cos \theta}{\Delta t} \]

or

\[ \frac{\Delta \Phi}{\Delta t} = B \cdot \frac{\Delta A}{\Delta t} \tag{1} \]

\[ \text{Magnitude of induced emf can be calculated by applying Faraday's law i.e.,} \]

\[ \mathbf{E} = N \frac{\Delta \Phi}{\Delta t} = N \cdot B \cdot \frac{\Delta A}{\Delta t} \tag{2} \]

Putting the values, we get;

\[ \mathbf{E} = 1 \times 0.6 \text{ T} \times 0.8 \text{ m}^2 \text{s}^{-1} \]

\[ \mathbf{E} = 0.48 \text{ V} \]

(P.T.O.)
15.4 LENZ'S LAW AND DIRECTION OF INDUCED EMF:

Statement: "The direction of the induced current is always so as to oppose the change which causes the current."

The Lenz's law refers to induced currents and not to induced emfs.

EXPLANATION.

According to Faraday's law (1831) of electromagnetic induction,

$$\mathbf{E} = -\mathbf{N} \frac{\mathbf{d} \Phi}{\mathbf{d}t}$$

The minus sign in the expression is very important.

To determine the direction, we use a method based on the discovery made by the Russian Physicist Heinrich Lenz in 1834. He found that the polarity of an induced emf always leads to an induced current that opposes, through the magnetic field of the induced current, the change inducing the emf. This rule is known as Lenz's law.

Lenz's law refers to induced currents and not to induced emfs, which means that we can apply it directly to close conducting loops or coils. However, if the loop is not closed, we can imagine as if it were closed, and then from the direction of induced current, we can find the direction of the induced emf.

Let us apply the Lenz law to the coil in which current is induced by the movement of a bar magnet. We know that a current carrying coil produces a magnetic field similar to that of a bar magnet. One face of the coil acts as the north pole while the other one as the south pole. If the coil is to oppose the
motion of the bar magnet, the face of the coil towards the magnet must become a north pole (fig 1). The two north poles will then repel each other. The right hand rule applied to the coil suggests that the induced current must be anticlockwise as seen from the side of the bar magnet.

According to Lenz law, the push of the magnet is the "change" that produces the induced current, and the current acts to oppose the push. On the other hand, if we pull the magnet away from the coil the induced current will oppose the "pull" by creating a south pole on the face of coil towards the bar magnet.

**LENZ LAW AND LAW OF CONSERVATION OF ENERGY.**

The Lenz's law is also a statement of law of conservation of energy that can be conveniently applied to the circuits involving induced currents. This can be proved with the help of an experiment.

**EXPERIMENT.**

The experimental arrangement is shown in fig 2. When the rod moves towards right, emf is induced in it, and an induced current flows through the loop.
in the anti-clockwise direction.

Since the current carrying rod is moving in the magnetic field, it experiences a magnetic force \( \mathbf{F} \); its magnitude is given by:

\[ F = ILB \sin \theta \]

By right hand rule the direction of \( F_m \) is opposite to that of \( \mathbf{V} \) so it tends to stop the rod.

[Fig 2(a)]. An external dragging force equal to \( F_m \) in magnitude but opposite in direction must be applied to keep the rod moving with constant velocity.

This energy is the source of induced current. Thus electromagnetic induction is exactly according to law of conservation of energy.

The Lenz law forbids the induced current directed clockwise in this case, because the force \( F_m \) would be, then in the direction of \( \mathbf{V} \) that would accelerate the rod towards right.

[Fig 2(b)]. This in turn would induce a stronger current, the magnetic field due to it also increases and the magnetic force increases further. Thus, the motion of the wire is more accelerated and so on. Commencing with a minute quantity of energy, we obtain an ever increasing K.E of motion apparently from nowhere. Thus the process becomes self-perpetuating which is against the law of conservation of energy.

(P.T.O.)
# 15.5 MUTUAL INDUCTION:

Def.: The phenomenon in which a changing current in one coil induces an emf in another coil is called the mutual induction.

EXPLANATION.

Consider two coils placed close to each other as shown in the fig. The primary coil is connected with a battery through a switch and a rheostat, and the secondary coil is connected to a galvanometer. If the current in the primary coil is changed with time by varying the resistance of the rheostat, the magnetic flux of the primary coil also changes with time. But the secondary coil is in the magnetic field of the primary coil, therefore any change in the magnetic field of the primary coil causes an induced emf in the secondary coil.

According to Faraday's law of electromagnetic induction, the emf induced in the secondary coil is proportional to the rate of change of flux passing through it. Therefore,

\[ E_s = -N_s \frac{d\Phi}{dt} \]

where \( N_s \) is the number of turns in the secondary coil. Let the flux passing through one loop of the secondary coils be \( \Phi \). Net flux passing through the coil of \( N_s \) loops is \( N_s \Phi \). As this flux is proportional to the magnetic field produced by the current Ip in the
primary and the magnetic field itself is proportional to {\( I_p \)} so as to:

\[ \phi_s \propto B \]

and \( B \propto I_p \)

\[ \therefore \phi_s \propto I_p \]

or:

\[ N_2 \phi_s \propto I_p \]  

where \( M \) is proportionality constant called the mutual inductance of the two coils.

Putting the value of \( N_2 \phi_s \) from eq. 2 in eq. 1, we have:

\[ E_s = -M \frac{\Delta I_p}{\Delta t} \]

or:

\[ E_s = -M \frac{\Delta I_p}{\Delta t} \]  

This eq. shows that changing the current \( I_p \) in the primary coil with time induces an emf in the secondary coil. It means that the emf induced in the secondary coil is proportional to the change of current in the primary coil.

The negative sign in eq. 2 indicates that the induced emf is in the opposite direction to the change of current in the primary coil. From eq. 2, we have:

\[ M = \frac{E_s}{\Delta I_p/\Delta t} \]  

(Numerically)

MUTUAL INDUCTANCE.

Definition: “The ratio of average emf induced in the secondary coil to the change of current with time in the primary coil is called mutual inductance.”

UNIT. The SI unit of mutual inductance is V·s·A⁻¹, which is called henry (H) after Joseph Henry.

HENRY. “Two coils are said to have a mutual induction of 1 henry if an emf of 1 volt is induced in the secondary coil when the current in the primary coil changes at the rate of 1 ampere per second.”

(P.T.O.)
Dependence of Mutual Induction.

Mutual induction of two coils depends upon the following factors:

1. Number of turns of the coils
2. Area of cross-section of the coils
3. Closeness of the coils
4. Nature of the core material upon which the two coils are wound.

**EXAMPLE 15.3** An emf of 5.6 V is induced in a coil while the current in a nearby coil is decreased from 100 A to 20 A in 0.02 s. What is the mutual inductance of the two coils? If the secondary has 200 turns, find the change in flux during this interval.

**DATA.**
- emf induced in the secondary coil: \( E_s = 5.6 \text{ V} \)
- change in current in primary coil: \( \Delta I_p = 100 \text{ A} - 20 \text{ A} = 80 \text{ A} \)
- Time interval for the change: \( \Delta t = 0.02 \text{ s} \)
- No. of turns in the secondary: \( N_s = 200 \)
- Mutual inductance: \( M = ? \)
- Change in flux: \( \Delta \phi = ? \)

**Sol.** Using the rel. for mutual inductance;

\[
M = \frac{E_s}{\Delta t} = \frac{5.6 \text{ V}}{80 \text{ A} / 0.02 \text{ s}} = 1.4 \times 10^{-4} \text{ H}
\]

According to Faraday's law, we have:

\[
E_s = N_s \frac{\Delta \phi_s}{\Delta t}
\]

or:

\[
\phi_s = \frac{N_s E_s \Delta t}{\Delta \phi_s}
\]

\[
\phi_s = \frac{5.6 \text{ V} \times 0.02 \text{ s}}{200} = 5.6 \times 10^{-4} \text{ Wb.}
\]

\( (\text{P.T.S}) \)
# 15.6 SELF INDUCTION:

Def. "The phenomenon in which a changing current in a coil induces an emf in itself is called self-induction, or self-inductance \( L \) of a coil is the ratio of the average emf to the rate of change of current in the coil."

EXPLANATION.

Consider a coil connected in series with a battery and a rheostat as shown in the fig. (1). When a current passes through the coil, magnetic flux is produced. If the current is changed by varying the rheostat quickly, magnetic flux through the coil changes that causes an induced emf in the coil. Such an emf is called as self induced emf.

If the magnetic flux through one loop of the coil is \( \Phi \) and \( N \) are the total number of turns of the coil, then total flux through the coil will be

\[
\text{Total flux} = \Phi \times N
\]

As magnetic flux \( \Phi \) is proportional to the magnetic field which is in turn proportional to the current \( I \) (i.e., \( \Phi \propto B; B \propto I \)), therefore

\[
N \Phi \propto I \quad \text{or} \quad N \Phi = L \cdot I \quad \text{(2)}
\]

where \( L \) is the constant of proportionality called the self-inductance of the coil.

By Faraday's law, emf induced in the coil is

\[
E_L = -N \frac{\Delta \Phi}{\Delta t}
\]

or \( E_L = -\frac{\Delta (N \Phi)}{\Delta t} \quad \text{(3)} \)

Using eq. (2) in eq. (3), we have

\[
E_L = -L \frac{\Delta I}{\Delta t} = -L \frac{DI}{Dt}
\]

(\( F.T.O. \))
i.e; self induced emf in the coil is proportional to the rate of current in the coil.

From eq (4);

\[ L = \frac{\varepsilon}{\frac{di}{dt}} \]  

\[ (\text{numerically}) \]  

Def. "The ratio of the average emf to the rate of change of current in the coil is called self inductance."

UNIT. The SI unit of self inductance is V S A^{-1}, which is called a henry (H) after Joseph Henry.

Dependence of Self Inductance.

It depends upon the following factors:

1. No. of turns of the coil
2. Area of cross section of the coil
3. The core material.

FERROMAGNETIC CORE.

By winding the coil around a ferromagnetic (iron) core, the magnetic flux and hence the inductance can be increased significantly relative to that for an air core.

INDUCED EMF IN SELF INDUCTANCE AS BACK EMF.

The negative sign in eq (4) indicates that the self induced emf must oppose the change that produced it. That is why the self induced emf is sometimes called as back emf. This is exactly in accord with the Lenz’s law.

If the current is increased, the induced emf will be opposite to that of battery and if the current is decreased the induced emf will aid, rather than oppose the battery.

INDUCTOR.

"A circuit element in which a self induced emf accompanies a changing circuit is called an inductor."
Because of their self-inductance, coils of wire are known as inductors and are widely used in electronics. In a.c., inductors behave like resistors.

**EXAMPLE 15.4:** The current in a coil of 1000 turns is changed from 5A to zero in 0.25 s. If an average emf of 50 V is induced during this interval, what is the self-inductance of the coil? What is the flux through each turn of the coil when a current of 6 A is flowing?

**DATA**
- Change in current $\Delta I = 5A - 0 = 5A$
- Time interval $\Delta t = 0.25 s$
- emf induced $E_L = 50 V$
- Steady current $I = 5 A$
- No. of turns of coil $N = 1000$
- Self inductance $L = ?$
- Flux through each turn $\Phi = ?$

**Sol.:**

As $E_L = L \frac{\Delta I}{\Delta t}$ (Numerically)

or $L = \frac{E_L}{\frac{\Delta I}{\Delta t}}$  \(\text{eqn. 1}\)

Putting the values, we have,

$L = \frac{50V}{5A/0.25s} = 2 \text{ H}$

Now using the relation:

$N\Phi = LI$

or $\Phi = \frac{LI}{N}$  \(\text{eqn. 2}\)

Putting the values, we have:

$\Phi = 2 \text{ H} \times 6A = \frac{12 \text{ Wb}}{1000} = 1.2 \times 10^{-3} \text{ Wb}$

(P.T.O.)
15.7 ENERGY STORED IN AN INDUCTOR:

The magnetic energy in an inductor may be compared with the electrostatic energy in a capacitor. When a capacitor is charged, electric P.E. is stored in the electric field between the capacitor plates; the energy is released as the capacitor discharges and the electric field is reduced. When current is produced in an inductor, electrical P.E. is converted to magnetic energy, which is stored in the magnetic field inside the inductor. This energy is released when the inductor's current and its associated magnetic field decrease.

EXPRESSION FOR ENERGY IN AN INDUCTOR.

Consider a coil connected to a battery and a switch in series as shown in Fig. When the switch is turned on, voltage $V$ is applied across the ends of the coil and current $I$ through it rises from zero to max. value $I_m$. Due to change of current $I$, an emf is induced, which is opposite to that of battery. Work is done by the battery to move charges against the induced emf.

Work done by the battery in moving a small charge $dq$ is

$$\Delta W = dq \cdot E_L$$  \hspace{1cm} (1)

where $E_L$ is the magnitude of induced emf given by

$$E_L = L \cdot \frac{dI}{dt}$$  \hspace{1cm} (2)

Putting this value of eq. (2) in eq. (1), we get

$$W = dq \cdot L \frac{dI}{dt}$$  \hspace{1cm} (3)

As the current increases $dt$ from zero to a max. value $I_m$, more and more energy is stored in the inductor. Therefore, total work done by the battery is found

(P.T.O.)
by inserting the change in current, $\Delta I$, and the average current, $\frac{\Delta q}{\Delta t}$ in eq. (3), we get:

$$\text{Average current} = \frac{\Delta q}{\Delta t} = \frac{q+I}{2} \Rightarrow \frac{\Delta q}{\Delta t} = \frac{\Delta I}{2}$$

and change in current $= LI = I - 0 = I$

Therefore, eq. (3) becomes:

$$\text{Total work} = W = \left(\frac{1}{2} I\right) LI$$

$$W = \frac{1}{2} LI^2$$

This work is stored as P.E in the inductor. Hence, the energy stored in an inductor is:

$$U_m = \frac{1}{2} LI^2$$

i.e., the energy stored in the inductor at any instant depends on the current in the inductor at that instant.

This expression has the same form as the eq. for the electrostatic energy stored by a capacitor $C$ with a voltage $V$ across its plates i.e.,

$$\text{Energy} = \frac{1}{2} CV^2.$$

**ENERGY STORED IN AN INDUCTOR IN TERMS OF THE MAGNETIC FIELD.**

As in case of a capacitor, energy is stored in the electric field between the plates and likewise in an inductor, energy is stored in the magnetic field. Therefore, eq. (5) can be expressed in terms of the magnetic field $B$, for the special case of a long, thin **solenoid**. Let $n$ be the number of turns per unit length of the coil of the solenoid and area of cross section $A$.

The magnetic field strength inside the solenoid is $B = \mu_0 n I$

Since, flux through the coil $= \phi = BA$

so,

$$\phi = \mu_0 n I A$$

As we know,

$$N \phi = LI$$

or

$$L = \frac{N \phi}{I}$$
Putting the value of $\Phi$ from eq. 5 into eq. 4, we have

$$L = \frac{N}{I} \cdot \mu_n I A$$

or

$$L = N \mu_n A$$

.... \[8\]  

$$n = \frac{N}{\ell}$$

If $\ell$ is the length of the solenoid, then putting $N=n\ell$ in eq. 8, we have the self-inductance of the solenoid as

$$L = (n\ell) \mu_n A$$

.... \[9\]

Now, substituting $\ell$ for $L$ of eq. 9 in eq. 5, we have

$$U_m = \frac{1}{2} \cdot (\mu_n A \ell) I^2$$

.... \[10\]

Since the magnetic field for a solenoid is given by

$$B = \mu_n I$$

or

$$I = \frac{B}{\mu_n}$$

Putting this value in eq. 10, we have

$$U_m = \frac{1}{2} \cdot (\mu_n A \ell) \cdot \left(\frac{B}{\mu_n}\right)^2$$

$$= \frac{1}{2} \cdot \mu_n A \ell \cdot \frac{B^2}{\mu_n^2 \ell^2}$$

$$= \frac{1}{2} \cdot \frac{\mu_n A \ell \cdot B^2}{\mu_n^2 \ell^2}$$

or

$$U_m = \frac{1}{2} \cdot \frac{B^2}{\mu_n} \cdot (A \ell)$$

.... \[12\]

**ENERGY DENSITY.**

Def: "The energy stored per unit volume inside the solenoid is called energy density."

Therefore, dividing eq. 12 by volume $(A \ell)$, we get

energy density:

$$U_m' = \frac{U_m}{(A \ell)} = \frac{1}{2} \cdot \frac{B^2}{\mu_n} \cdot \frac{A \ell}{A \ell}$$

$$U_m' = \frac{1}{2} \cdot \frac{B^2}{\mu_n}$$

(P.T.O.)
**EXAMPLE 15.5:** A solenoid coil 10.0 cm long has 40 turns per cm. When the switch is closed, the current rises from zero to its max. value of 5 A in 0.01 s. Find the energy stored in the magnetic field if the area of cross-section of the solenoid be 28 cm².

**DATA:** Length of solenoid = \( l = 10.0 \text{ cm} = 0.1 \text{ m} \)

No. of turns = \( n = 40 \text{ per cm} = 4000 \text{ per m} = 4000 \text{ m}^{-1} \)

Steady current = \( I = 5 \text{ A} \)

Area of cross-section = \( A = 28 \text{ cm}^2 = 28 \times 10^{-4} \text{ m}^2 \)

Time interval = \( \Delta t = 0.01 \text{ s} \)

Energy stored = \( U_m = ? \)

**Sol.:** 

As \( E = \frac{1}{2} L I^2 \quad \text{(1)} \)

but \( L = \frac{\mu_0 n^2 A l}{2} \quad \text{(2)} \)

Putting the values, we have

\[
L = 4\pi \times 10^{-7} \text{ Wb m}^{-1} \times (4000 \text{ m}^{-1}) \times 28 \times 10^{-4} \text{ m} \times 0.1 \text{ m} \\
L = 5.63 \times 10^{-3} \text{ H} \quad \text{(3)}
\]

Now, putting the value in eq. (1), we have

\[
U_m = \frac{1}{2} \left( 5.63 \times 10^{-3} \text{ H} \right) \times (5 \text{ A})^2 \\
U_m = 1.64 \times 10^{-2} \text{ J}
\]

**FOR YOUR INFORMATION:**

An induction ammeter with its iron core jaw (a) open and (b) closed around a wire carrying an alternating current \( I \).

Some of the magnetic field lines that encircle the wire are routed through the coil by the iron core and lead to an induced emf. The meter detects the emf and is calibrated to display the amount of current in the wire.

(P.T.O.)
# 15.8 ALTERNATING CURRENT GENERATOR

Def: "A current generator is a device which converts mechanical energy into electrical energy and generates alternating emf and current."

PRINCIPLE.

The principle of an electric generator is based on Faraday's law of electromagnetic induction. When a coil is rotated in a magnetic field by some mechanical means, magnetic flux through the coil changes, and consequently an emf is induced in the coil.

CONSTRUCTION.

An A.C. generator consists of the following components:

1. Armature (which consists of a coil of many turns wound on a core of high permeability and loss hysteresis is low)
2. Magnetic field (which may be due to permanent magnet or an electromagnet)
3. Slip rings (which are concentric with the axis of the loop and rotate with it)
4. Carbon brushes (slip rings slide against carbon brushes to which external circuit is connected)

WORKING.

Consider a rectangular loop of wire of area $A$ be placed in a uniform magnetic field $B$. The loop is rotated about z-axis through its centre at constant angular velocity $\omega$. One end of the loop is attached to a metal ring $R$ and the other end to the ring (P-T-S)
The rings, called the slip rings, are equidistant with the axis of the loop and rotate with it. Rings RR slide against stationary carbon brushes to which external circuit is connected.

**Expression for Induced EMF and Current.**

Consider the position of fig.(2), while it is rotating anticlockwise. The vertical side ab of the loop is moving with velocity $\mathbf{v}$ in the magnetic field $\mathbf{B}$. If the angle between $\mathbf{v}$ and $\mathbf{B}$ be $\theta$, the motional emf induced in the side ab has the magnitude

$$E_{ab} = VBL \sin \theta$$  \hspace{1cm} (1)

The direction of induced current in the wire ab is the same as that of force $\mathbf{F}$ experienced by the positive charges in the wire, i.e., from top to the bottom.

The same amount of emf is induced in the side cd, but the direction of current is from bottom to the top. Therefore,

$$E_{cd} = VBL \sin \theta$$  \hspace{1cm} (2)

The net contribution to emf by sides bc and da is zero because the force acting on the charges inside bc and da is not along the wire. Thus

$$E_{bc} = E_{da} = 0$$

Since both the emfs in the sides ab and cd drive current in the same direction around the loop, the total emf in the loop is

$$E = E_{ab} + E_{cd}$$

Substituting the values from eqs (1) and (2), we have

$$E = VBL \sin \theta + VBL \sin \theta$$

or

$$E = 2VBL \sin \theta$$  \hspace{1cm} (3)

(P.I.O)
If the loop is replaced by a coil of $N$ turns, the total emf in the coil will be

$$E = 2NBLv \sin \theta \quad \text{(4)}$$

The linear speed $v$ of the vertical wire is related to the angular speed $\omega$ by the relation

$$v = R\omega$$

where $R$ is the distance of the vertical wires from the center of the coil. Substituting $R\omega$ for $v$ in eq. (4), we get

$$E = 2N(\omega R)BL\sin \theta$$

$$E = N\omega (2RL) B \sin \theta \quad \text{(5)}$$

where $A = 2RL$ is the area of the coil.

As the angular displacement $\theta = \omega t$, so eq. (5) becomes;

$$E = N\omega AB \sin (\omega t) \quad \text{(6)}$$

i.e., induced emf varies sinusoidally with time. It has the maximum value when $\sin(\omega t)$ is equal to 1.

Thus

$$E_0 = N\omega AB \quad \text{(7)}$$

so eq. (6) can be written as;

$$E = E_0 \sin(\omega t) \quad \text{(8)}$$

If $R$ is the resistance of the coil, then by Ohm's law, the induced current $I$ in the coil will be

$$I = \frac{E}{R} = \frac{E_0 \sin(\omega t)}{R} = \frac{E_0}{R} \sin(\omega t)$$

or

$$I = I_0 \sin(\omega t) \quad \text{(9)}$$

Angular speed $\omega$ of the coil is related to its freq. of rotation $f$ as

$$\omega = 2\pi f$$

so eq. (8) and (9) can be written as:

$$E = E_0 \sin(2\pi ft) \quad \text{(10)}$$

$$I = I_0 \sin(2\pi ft) \quad \text{(11)} \quad \text{(P.T.O.)}$$
VARIATION OF CURRENT AS A FUNCTION OF $\theta$

Eq. (1) indicates the variation of current as a function of $\theta = 2\pi t$. Fig. 4 shows the graph for the current corresponding to different positions of one loop of the coil.

(i) When the angle between $\vec{V}$ and $\vec{B}$ is $0^\circ$, then the plane of the loop is perpendicular to $\vec{B}$ and current will be zero.

(ii) As $\theta$ increases, current also increases and at $\theta = 90^\circ = \frac{\pi}{2}$ rad, the loop is parallel to $\vec{B}$, current is max, directed along $\overrightarrow{abcda}$.

(iii) On further increase, in $\theta$ current decreases and at $\theta = 180^\circ = \pi$ rad the current becomes zero as the loop is again perpendicular to $\vec{B}$.

(iv) For $180^\circ < \theta < 270^\circ$, current increases but reverses direction, as is clear from the fig. (4). Current is now directed along $\overrightarrow{eabcd}$. At $\theta = 270^\circ = \frac{3\pi}{2}$, current is max in the reverse direction as the loop is parallel to $\vec{B}$.

(v) At $\theta = 360^\circ = 2\pi$ rad, one rotation is completed, the loop is again perpendicular to $\vec{B}$ and the current decreases to zero.

(P.T.O.)
one rotation the cycle repeats itself. The current
alternates in direction once in one cycle. Therefore,
such a current is called the alternating current,
which reverses in direction \( f \) times per second.

In actual practice a number of coils are
wound around an iron cylinder which is rotated in
the magnetic field. This assembly is called an armature.
The magnetic field is usually provided by an
electromagnet. Armature is rotated by a fuel engine
or a turbine run by a waterfall. In some commer-
cial generators, field magnet is rotated around a
stationary armature.

**EXAMPLE 15.6:** An alternating current generator
operating at 50 Hz has a coil of 200 turns. The
coil has an area of 120 cm\(^2\). What should be the
magnetic field in which the coil rotates in order to
produce an emf of max value of 240 volt?

**DATA**

- Frequency of rotation \( f = 50 \) Hz
- No. of turns of the coil \( N = 200 \)
- Area of the coil \( A = 120 \text{ cm}^2 = 1.2 \times 10^{-2} \text{ m}^2 \)
- Max. value of emf \( E_0 = 240 \text{ V} \)
- Max. flux density \( B = \) ?

**Sol.** Max value of emf is given by the rels;

\[
E_0 = N \omega AB \quad (1)
\]

or

\[
E_0 = N \times 2\pi f \times AB \quad (2) \quad \text{or} \quad \omega = 2\pi f
\]

or

\[
B = \frac{E_0}{2\pi f NA}
\]

Putting the values, we have

\[
B = \frac{240 \text{ V}}{2 \times 3.14 \times 50 \text{ Hz} \times 200 \times 1.2 \times 10^{-2} \text{ m}^2}
\]

\[
B = \frac{240 \text{ V}}{0.32 \text{ T} = 0.32 \text{ V s rad}^{-1} \text{ m}^{-2}}
\]

(P.T.O)


# 15.9 D C GENERATOR

A C generators are not fit for many applications, for example, to run a d.c motor. In 1834, William Sturgeon invented simple device called a commutator that prevents the direction of the current from changing. Def — "It is a device which is used to produce direct emf (direct current) called d.c generator."

CONSTRUCTION.

The construction of d.c generator is similar to that of an a.c generator (i.e., magnet, armature, carbon brushes etc) with the difference that the slip rings are replaced by commutators as shown in fig(1), which are simply split rings, A and A', each half being connected to each end of the coil. The brushes touch against the commutators.

WORKING.

Due to the flow of current, a magnetic field is produced in which the coil is rotated, and an induced emf is produced in it. When the current is zero and is about to change direction, the split rings also change the contacts with the carbon brushes BB as shown in fig(1). In this way the output from BB remains in the same direction, although the current is not constant in magnitude. The curve of the circuit is shown in fig(2). It is similar to the sine curve with the lower half inverted.

(P.T.O.)
REDUCTION OF OUTPUT WAVEFORM FLUCTUATION IN D.C. GENERATOR.

The fluctuations of the output waveform can be significantly reduced by using many coils rather than a single one. Multiple coils are wound around a cylindrical core to form the armature. Each coil is connected to a separate commutator and the output of every coil is tapped only as it reaches its peak emf. Thus the emf in the outer circuit is almost constant.

# 15.10 BACK MOTOR EFFECT IN GENERATORS:

In order to run a turbine, the shaft of the turbine is attached to the coil which rotates in a magnetic field. It converts the mechanical energy of the driven turbine to electrical energy. The generator supplies current to the external circuit.

LOAD: The devices in the circuit that consume electrical energy are known as the "Load". The greater the load, the larger the current is supplied by the generator. When the circuit is open, the generator does not supply electrical energy, and a very little force is needed to rotate the coil.

As soon as the circuit is closed, a current is drawn through the coil. The magnetic field exerts force on the current carrying coil.
BACK MOTOR EFFECT.

Fig (b) shows the forces acting on the coil. Force $\mathbf{F}_1$ is acting on the left side of the coil whereas an equal but opposite force $\mathbf{F}_2$ acts on the right side of the coil. These forces are such that they produce an anti-clockwise torque that opposes the rotational motion of the coil. This effect is sometimes known as back motor effect in the generators. The larger the current drawn, the greater is the counter torque produced. That means more mechanical energy is required to keep the coil rotating with constant angular speed. This is in agreement with the law of conservation of energy.

\# 15.11 DC MOTOR:

Def: "A motor is a device which converts electrical energy into mechanical energy.

i.e., A motor is just like a generator running in reverse. Electrical motors form the heart of a whole host of devices ranging from domestic appliances such as vacuum cleaners and washing machines to electric trains and lifts.

PRINCIPLE:

Its basic principle is the fact that a current carrying conductor placed in a magnetic field experiences a force.

CONSTRUCTION:

A D.C motor is similar in construction to a generator, having a magnetic field, a commutator and an armature. Instead of rotating its coil (armature), electric current is fed into the coil which then rotates. Fig. shows a D.C motor which

(P.T.O)
consists of a coil capable of rotating in a magnetic field and fitted with a commutator.

**WORKING.**

When current flows in the armature coil, the force on the conductors produces a torque that rotates the armature. The amount of this torque depends upon the current, the strength of the magnetic field, the area of the coil, and the number of turns of the coil.

If the current in the coil remains in the same direction, the torque on it would reverse after each half revolution, at this moment commutator is used to reverse the current in each coil at the proper instant to produce the torque always in the same direction.

**SMOOTH RUNNING.**

A little problem arises due to the use of commutator. That is, the torque vanishes each time the current changes its direction. This creates jerks in the smooth-running of the armature. However, the problem is overcome by using more than one coils wrapped around a soft iron core. This results in producing a more steady current.

(P.T.O)
#15.12 BACK EMF EFFECT IN MOTORS:

When the coil of the motor rotates across the magnetic field, an emf, \( E \), is induced in it. The induced emf is in such a direction to oppose the emf running the motor. Due to this reason the induced emf is called back emf of the motor. The magnitude of the back emf increases with the speed of the motor.

Since \( V \) and \( E \) are opposite in polarity, the net emf in the circuit is \( V - E \). If \( R \) is the resistance of the coil and \( I \) the current drawn by the motor, then by Ohm's Law:

\[
I = \frac{V - E}{R}
\]

or

\[
V = E + IR \tag{1}
\]

When the motor is just started, back emf is almost zero and hence a large current passes through the coil. As the motor speeds up, the back emf increases and the current becomes smaller and smaller. However, the current is sufficient to provide torque on the coil to drive the load and to overcome losses due to friction.

**OVERLOADED.**

If the motor is overloaded, it slows down. Consequently, the back emf decreases and allows the motor to draw more current. If the motor is overloaded beyond its limits, the current could be so high that it may burn the motor out.

(P.T.O.)
EXAMPLE 15.7: A permanent magnet D.C. motor is run by a battery of 24 volts. The coil of the motor has a resistance of 2 ohms. It develops a back emf of 22.5 volts when driving the load at normal speed. What is the current when motor just starts up? Also find the current when motor is running at normal speed.

**DATA.**
- Operational voltage \( V = 24 \) volt
- Resistance of the coil \( R = 2 \) ohm
- Back emf \( E = 22.5 \) volt
- Current \( I = ? \)

**sol. (i)** When motor just starts up, the back emf \( E = 0 \).

Thus, using the relation;

\[ V = E + IR \]

\[ 24 = 0 + I \times 2 \]

\[ I = \frac{24}{2} = 12 \text{ A} \]

(ii) When motor runs at normal speed, the back emf \( E = 22.5 \) volt. Again using the relation;

\[ V = E + IR \]

\[ 24 = 22.5 + I \times 2 \]

\[ I = \frac{24 - 22.5}{2} = 0.75 \text{ A} \]

**# 15.13 TRANSFORMER:**

**Def.** A transformer is an electrical device used to change a given alternating emf into a larger and smaller alternating emf.

**CONSTRUCTION.**

The transformer consists of two coils of copper electrically insulated from each other wound on the same iron core. The coil to which A.C. power is supplied.
is called **primary** and that from which power is
delivered to the circuit is called the **secondary**.

There is no electrical connection between the two
coils, but they are magnetically linked.

**PRINCIPLE.**

The transformer works on the principle of
**mutual induction** between the two coils. A changing
current in the primary induces an emf in the secondary.

**WORKING.**

Suppose that an alternating emf is supplied to
the primary. If at some instant \( \frac{\Delta \phi}{\Delta t} \), the flux in the
primary is changing at the rate of \( \frac{\Delta \phi}{\Delta t} \), then there
will be back emf induced in the primary which will
oppose the applied voltage. The instantaneous value
of the self-induced emf is given by:

\[
\text{Self induced emf} = -N_p \left( \frac{\Delta \phi}{\Delta t} \right)
\]

If the resistance of the coil is negligible then the back
emf is equal and opposite to applied voltage \( V_p \). It means:

\[ V_p = - \text{back emf} = N_p \left( \frac{\Delta \phi}{\Delta t} \right) \quad (1) \]

where \( N_p \) is the number of turns in the primary coil.

Assuming the flux through the primary also
passes through the secondary i.e. the two coils are
tightly coupled, the rate of change of flux in the
secondary will also be \( \frac{\Delta \phi}{\Delta t} \) and the magnitude of
the induced emf across the secondary is given by:

\[ V_s = N_s \left( \frac{\Delta \phi}{\Delta t} \right) \quad (2) \]

where \( N_s \) is the number of turns in the secondary coil.

Dividing eq. (2) by (1), we have:

\[ \frac{V_s}{V_p} = \frac{N_s}{N_p} \quad (3) \]

\( (P-T-s) \)
TYPES OF TRANSFORMER.

1. Step-up transformer
2. Step-down transformer

(1) Step-up Transformer

When \( N_s > N_p \), i.e., the number of turns in the secondary is greater than the primary, then \( V_s > V_p \), so step-up transformer is defined as:

"Such a transformer in which voltage across secondary is greater than the primary is called step-up transformer."

(2) Step-down Transformer

If \( N_s < N_p \), i.e., the number of turns in the secondary are less than the number of turns in the primary, then \( V_s < V_p \), thus step-down transformer can be defined as:

"Such a transformer in which voltage across secondary is less than the primary voltage is called a step-down transformer."

Transformer symbol is shown in fig (4).

**ECONOMICAL TRANSMISSION OF A.C. OVER LONG DISTANCES.**

The electrical power in a transformer is transformed from its primary to the secondary coil by means of changing flux. For an ideal case, the power input
The primary is nearly equal to the power output from the secondary i.e;

\[ V_p I_p = V_s I_s \]

Or, \[ \frac{V_s}{V_p} = \frac{I_r}{I_s} \quad (4) \]

\( I_p \) is the current in the primary and \( I_s \) in the secondary. The currents are thus inversely proportional to the respective voltages. Therefore, in a step-up transformer when the voltage across the secondary is raised, the value of current is reduced. This is the principle behind its use in the electric supply network where transformer increases the voltage and reduces the current so that it can be transmitted over long distances without much power loss. When current \( I \) passes through a resistance \( R \), economically then the power loss due to heating effect is \( I^2R \).

In order to minimize the loss during transmission, it is not possible to reduce \( I \) because it requires the use of thick copper wire which becomes highly uneconomical. So at the generating power station, the voltage is stepped up to several thousand of volts and power is transmitted to long distances without much loss. Step-down transformer then decrease the voltage to a safe value at the end of line where the consumer of electric power is located. Inside a house, a transformer may be used to step-down the voltage from 250 volts to 7 volts for ringing bell or operating a radio. Transformer with several secondaries are used in the television and radio receivers where several different voltages are required.

(P.T.O)
IDEAL AND ACTUAL TRANSFORMERS.

In an ideal transformer, the output power is nearly equal to the input power. But in an actual transformer, it could not happen. The output is always less than input due to power losses.

DIFFERENT CAUSES OF POWER LOSS.

There are two main causes of power loss, which are:

1. Eddy Currents
2. Magnetic Hysteresis

1. EDDY CURRENT OR FOUCAULT CURRENT.

Def: "An electric current induced within the body of a conductor when that conductor either moves through a non-uniform magnetic field or in a region where there is a change in magnetic flux is known as eddy current."

In order to enhance the magnetic flux, the primary and secondary coils of the transformer are wound on soft iron core. The flux generated by the coils also passes through the core. As magnetic flux changes through a solid conductor, induced currents are set up in closed paths in the body of the conductor. These induced currents are set up in a direction perpendicular to the flux changes and are known as eddy currents. It results in power dissipation and heating of the core material. In order to minimize the power loss due to flow of these currents, the core is laminated with insulation in between the layers of laminations which stops the flow of eddy currents.

2. MAGNETIC HYSTERESIS.

Def: "Lagging of changes in the magnetization of a substance behind changes in the magnetic field as the magnetic field is varied is known as magnetic hysteresis."
Hysteresis loss is the energy expended to magnetize and demagnetize the core material in each cycle of the A.C.

**EFFICIENCY OF A TRANSFORMER.**
The efficiency of a transformer can be defined as

\[ E = \frac{\text{Output power}}{\text{Input power}} \times 100 \]

**IMPROVEMENT IN EFFICIENCY.**
In order to improve the efficiency, to minimize all the power losses:

1. **Loop area.**
   Core should be assembled from the laminated sheets of a material whose hysteresis loop area is very small.

2. **Insulation.**
   The insulation between lamination sheets should be perfect so as to stop the flow of eddy currents.

3. **Resistance of coils.**
   The resistance of the primary and secondary coils should be kept to a minimum.

4. **Flux coupling.**
   As power transfer from primary to secondary takes place through flux linkages, so the primary and secondary coils should be wound in such a way that flux coupling between them is maximum.

(P.T.O)
**EXAMPLE 15.8:** The turn ratio of a step-up transformer is 50. A current of 20 A is passed through its primary coil at 220 volts. Obtain the value of the voltage and current in the secondary coil assuming the transformer to be ideal one.

**DATA**
- Turn ratio \( N_p = N_s = 50 \)
- Current in primary coil \( I_p = 20 \) A
- Voltage applied to primary coil \( V_p = 220 \) V
- Voltage in secondary coil \( V_s = ? \)
- Current in secondary coil \( I_s = ? \)

**Sol.** Using the relation:

\[
\frac{V_s}{V_p} = \frac{I_p}{I_s} \quad (1)
\]

Substituting the values, we get:

\[
\frac{V_s}{220} = 50 \quad \Rightarrow \quad V_s = 50 \times 220 \text{ V} = 11,000 \text{ V}
\]

From eq. (1), we have:

\[
I_s = \frac{V_p \times I_p}{V_s}
\]

Substituting the values, we have:

\[
I_s = \frac{220 \text{ V} \times 20 \text{ A}}{11,000 \text{ V}} = 0.4 \text{ A}
\]

(R.I.O.)
Chapter 15: Electromagnetic Induction

SHORT QUESTIONS.

Q.15.1. Does the induced emf in a circuit depend on the resistance of the circuit? Does the induced current depend on the resistance of the circuit?

Ans. The emf induced in a coil depends upon the rate of change of magnetic flux through the coil \( E = -\frac{d\Phi}{dt} \). Hence, its value does not depend upon the resistance of the coil. But the induced current flowing through a coil is equal to \( I = \frac{E}{R} \) and its value depends upon the resistance \( R \) of the coil. If the resistance increases, current will decrease because the product of \( I \) and \( R \) is always constant, i.e., \( I \cdot R = \text{constant} \).

Q.15.2. A square loop of wire is moving through a uniform magnetic field. The normal to the loop is oriented parallel to the magnetic field. Is a emf induced in the loop? Give a reason for your answer.

Ans. Induced emf will not be produced because there is no change in magnetic flux linking the loop i.e., \( \frac{d\Phi}{dt} = 0 \), so according to the relation \( E = -N \frac{d\Phi}{dt} = -N \cdot 0 = 0 \).

If the square loop is being rotated in the magnetic field in such a way that the loop is cutting the magnetic field lines due to its motion, then an emf will be produced in the coil.

Q.15.3. A light metallic ring is released from above into a vertical bar magnet. Viewed from above, does the current flow clockwise or anti-clockwise in the ring?

Ans. According to Faraday's Law, an induced emf and hence an induced current will be produced.
in the metallic ring. Lenz's law shows that the induced current in the ring should flow in the ring in such a way so as to oppose the cause producing it. The induced current flowing through the ring should produce a magnetic field that will oppose the motion to the loop toward the bar. So, the side of ring facing magnet must be north pole of the induced magnetic field. Right hand rule shows that the magnetic field will be produced only if induced current flows through the ring is clockwise direction.

Q 15.4: What is the direction of the current through resistor R as shown? As switch S is (a) closed (b) opened.

\textbf{Ans:} (a) When switch is closed, the current in the circuit increases from zero to max. steady value during this interval, magnetic flux in the second coil increases from zero to max., and an induced current is produced in it. The side of current carrying coil facing the other coil becomes north pole, so to oppose N-pole, the current in the other coil must flow anticlockwise. Hence, current in R flows from left to right as shown.

(b) However, when switch is opened, the current in circuit decrease from max. to zero and flux linked with other coil decreases and induced current is produced in reverse direction. So, current in R flows from right to left (clockwise) as shown.
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Q15.5: Does the induced emf always act to decrease the magnetic flux through a circuit?

Ans. No, the induced emf does not act so as to decrease the magnetic flux through a circuit. According to Lenz's law, the induced emf always acts so as to oppose the cause producing it. If an induced emf appears in a circuit due to decreasing magnetic flux linking that circuit, the induced current which flows through the circuit produces its own magnetic field that opposes the decrease of magnetic field. In other words, it is reinforcing or increasing the magnetic flux passing through the circuit.

Q15.6: When the switch in the circuit is closed, a current is established in the coil, and the metal ring jumps upward. Why? Describe what would happen to the ring if the battery polarity were reversed.

Ans. When switch in the circuit is closed, the current is set up in the coil. Magnetic flux changes through the metallic ring and an induced emf is produced in it. The face of the ring opposite to coil (according to Lenz's law) develops similar pole of magnet and experiences repulsion from the side of coil and jumps up. The same event occurs even if the polarity of the battery is reversed.

Q15.7: The figure shows a coil of wire in the xy-plane with a magnetic field directed along the y-axis. Around which of the three coordinate axes should the coil be rotated in order to generate an emf and a current in the coil?
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Ans. The coil must be rotated about the x-axis to get change of magnetic flux and an induced current through it.

Q 15.8: How would you position a flat loop of wire in a changing magnetic field so that there is no emf induced in the loop?

Ans. If the flat loop of wire is placed parallel to the changing magnetic field, no flux changes through it and hence no induced emf is produced.

Q 15.9: In a certain region the earth's magnetic field points vertically down. When a plane flies due north, which wing tip is positively charged?

Ans. In the vertically downward directed magnetic field, when an aeroplane flies in the direction of north, then the positive charge in the wing will experience a force \( \mathbf{F} = q (\mathbf{v} \times \mathbf{B}) \). Right hand rule shows that this force is in the direction of \( \mathbf{v} \times \mathbf{B} \) i.e. along the direction of west. Hence the left wing tip, pointing towards west is charged positively.

Q 15.10: Show that \( E \) and \( \frac{\Delta \Phi}{\Delta t} \) have the same units.

Ans. As \( E = \frac{W}{q} \), \( E \) has the unit volt (or J C\(^{-1}\)) \( \text{(1)} \). Moreover, \( \frac{\Delta \Phi}{\Delta t} \) has unit Wb s\(^{-1}\).

\[ Wb = \frac{N}{A} \times s = \frac{J}{C} = \text{Volt} \quad \text{(2)} \]

Hence both \( E \) and \( \frac{\Delta \Phi}{\Delta t} \) carry the same unit. \( \text{(F.T.10)} \)
Chapter 15: Electromagnetic Induction

Q 15.11: When an electric motor, such as an electric drill, is being used, does it also act as a generator? If so, what is the consequence of this?

Ans. When an electric motor is running, its armature is rotating in a magnetic field. A torque acts on the armature, and at the same time magnetic flux is changing through the armature which produces an induced emf. The induced emf opposes the rotation of armature. This means that motor also acts as a generator when it is running.

Q 15.12: Can a D.C. motor be turned into a D.C. generator? What changes are required to be done?

Ans. A D.C. motor can be converted into a D.C. generator. For this, the armature coil of the motor is to be coupled with some rotating body. The rotational motion of the body is transferred to the armature coil of the motor. Due to this rotation, the magnetic flux through the coil changes and so an emf will be induced at the output. The motor therefore becomes a generator.

Q 15.13: Is it possible to change both the area of the loop and the magnetic field passing through the loop and still, not have an induced emf in the loop?

Ans. Yes, if the plane of the loop is kept parallel to the direction of the magnetic field, no emf remains will be induced in the loop either by changing its area or by changing the magnetic field.

Q 15.14: Can an electric motor be used to drive an electric generator with the output from the generator being used to operate the motor?

Ans. No, it is not possible. Because if it is possible, it will be a self-operating system without getting energy from some external source and this is against the law of conservation of energy.

(P.T.O.)
Q 15.15: A suspended magnet is oscillating freely in a horizontal plane. The oscillations are strongly damped when a metal plate is placed under the magnet. Explain why this occurs?

**Ans.** Due to the oscillations of the magnet, the magnetic flux passing through the metallic plate changes. This produces an induced emf in the plate and hence an induced current. The induced current produces its own magnetic field which always opposes the motion of the bar magnet. Thus oscillatory motion of the bar magnet is therefore damped.

Q 15.16: Four unmarked wires emerge from a transformer. What steps would you take to determine the turns ratio?

**Ans.** With the help of an ohm metre first separate the wires into two pairs forming primary and secondary of the transformer. Then with one pair connect small a.c. voltage of known value treating this pair as the primary. With the help of voltmeter measure the voltage obtained at the other pair. So

\[ \frac{V_p}{V_s} = \frac{N_p}{N_s} \]

Q 15.17: (a) Can a step-up transformer increase the power level? (b) In a transformer, there is not transfer of charge from the primary to the secondary. How is then the power transferred?

**Ans.** (a) No, a step-up transformer can only increase the voltage level obtained at its secondary. It cannot increase the power level in the light of law of conservation of energy.

(b) Energy or power from primary coil is transferred to the secondary coil through the magnetic flux which is also linking the secondary coil.
Q. 15.8: When the primary of a transformer is connected to a.c. mains the current in it:
(a) is very small if the secondary circuit is open, but
(b) increases when the secondary circuit is closed.

Explain these facts.

Ans. (a) If the secondary circuit is open, the output power \( (V_1I_1) \) will be zero. Since output power of a transformer is always slightly smaller than the input power, so a very small current is drawn by the transformer from the a.c mains i.e; the input current \( (I_p) \) in primary coil is very small.

(b) When the secondary circuit is closed the output power increases. To produce this power, transformer will draw large current from the a.c. mains to increase its primary power \( (V_pI_p) \).
Chapter 15: Electromagnetic Induction

PROBLEMS

P. 15.1: An emf of 0.45 V is induced between the ends of a metal bar moving through a magnetic field of 0.22 T. What field strength would be needed to produce an emf of 1.5 V between the ends of the bar, assuming that all other factors remain the same?

DATA: 1st induced emf = E₁ = 0.45 V
Magnetic field = B₁ = 0.22 T
2nd induced emf = E₂ = 1.5 V

Magnetic field strength = B₂ = ?

**SOL.** We know that

\[ E₁ = B₁ V L \sin \theta \]  \hspace{1cm} (1)

\[ E₂ = B₂ V L \sin \theta \]  \hspace{1cm} (2)

According to given condition, \( V \), \( L \) and \( \theta \) do not change, so dividing eq. (1) by eq. (2), we have

\[ \frac{E₁}{E₂} = \frac{B₁}{B₂} \]

\[ B₂ = \frac{E₂}{E₁} \times B₁ \]

\[ B₂ = \frac{1.5 V \times 0.22 T}{0.45 V} = \frac{0.73 T}{0.45} \]

\[ B₂ = 1.62 T \] (P.T.O)
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P.15.2: The flux density $B$ in a region between the pole faces of a horseshoe magnet is $0.5 \text{ Wb m}^{-2}$ directed vertically downward. Find the emf induced in a straight wire 5.0 cm long, perpendicular to $B$ when it is moved in a direction at an angle of $60^\circ$ with the horizontal with a speed of $100 \text{ cm s}^{-1}$.

**DATA:** Flux density $B = 0.5 \text{ Wb m}^{-2}$ (vertically downward)
- Length of wire $L = 5.0 \text{ cm} = 0.05 \text{ m}$
- $\theta = 60^\circ$ with horizontal, $30^\circ$ with $B$ (vertical)
- Speed of wire $v = 100 \text{ cm s}^{-1} = 1.0 \text{ m s}^{-1}$
- Emf induced $E = ?$

**Sol.:** As $E = VB L \sin \theta$

$$E = \frac{1.0 \text{ m}^2 \times 0.5 \text{ Wb m}^{-2} \times 0.05 \text{ m} \times \sin 30^\circ}{1.25 \times 10^2 \text{ V}^2}$$

P.15.3: A coil of wire has 10 loops. Each loop has an area of $1.5 \times 10^{-2} \text{ m}^2$. A magnetic field is perpendicular to the surface of each loop at all times. If the magnetic field is changed from $0.05 \text{T}$ to $0.06 \text{T}$ in $0.1 \text{ s}$, find the average emf induced in the coil during this time.

**DATA:**
- No. of loops of a coil $N = 10$
- Area of each loop $A = 1.5 \times 10^{-2} \text{ m}^2$
- Change in magnetic field $\Delta B = 0.06 \text{T} - 0.05 \text{T} = 0.01 \text{T}$
- Time interval $\Delta t = 0.1 \text{ s}$
- Emf induced $E = ?$

**Sol.:** The magnitude of the induced emf in the coil is:

$$E = \frac{N \Delta \Phi}{\Delta t} \quad (1)$$

As $\Delta \Phi = \Delta BA$

$$E = N \frac{\Delta (BA)}{\Delta t} \quad (2)$$

Putting the values, we have $T = \frac{\text{NA}^{-1} \text{ m}^{-1}}{0.1 \text{ s}}$

$$E = \frac{10 \times 0.01 \text{ T} \times 1.5 \times 10^{-2} \text{ m}^2}{0.1 \text{ s}}$$

$(P.T.O.)$
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P.15.4: A circular coil has 15 turns of radius 2 cm each. The plane of the coil lies at 40° to a uniform magnetic field of 0.2 T. If the field is increased to 0.5 T in 0.2 s, find the magnitude of the induced emf.

**DATA**
- Number of turns: \( N = 15 \)
- Radius of the coil: \( r = 2 \text{ cm} = 2 \times 10^{-2} \text{ m} \)
- Angle by which the plane of the coil: \( \theta = 40° \)
- Increase in magnetic field: \( \Delta B = 0.5 T - 0.2 T = 0.3 T \)
- Time interval: \( \Delta t = 0.2 \text{ s} \)

**Induced emf: \( E = ? \)**

**Solu.** The magnitude of induced emf is given by:

\[
E = N \frac{\Delta \phi}{\Delta t} \tag{1}
\]

But:

\[
\Delta \phi = \Delta B \cdot A = \Delta B \left( \pi r^2 \right) \cos 40° \tag{2}
\]

Putting the values, we have:

\[
E = 15 \times 0.3 T \times 3.14 \times (2 \times 10^{-2} \text{ m})^2 \times 0.756 \times \frac{0.2 \text{ s}}{0.2 \text{ s}} = 0.021 \text{ V}
\]

P.15.5: Two coils are placed side by side. An emf of 0.8 V is observed in one coil when the current is changing at the rate of 200 A s⁻¹ in the other coil. What is the mutual inductance of the coils?

**DATA**
- Emf induced in the one coil: \( E_s = 0.8 \text{ V} \)
- Rate of change of current: \( \frac{\Delta I}{\Delta t} = 200 \text{ A s}^{-1} \)

**Mutual inductance of the coil: \( M = ? \)**

**Solu.**

\[
M = \frac{E_s}{\frac{\Delta I}{\Delta t}} = \frac{0.8 \text{ V}}{200 \text{ A s}^{-1}} = 4 \times 10^{-3} \text{ H} = 4 \text{ mH}
\]

(P.T.O.)
Chapter 15: Electromagnetic Induction

P.15.6 A pair of adjacent coils has a mutual inductance of 0.75 H. If the current in the primary changes from 0 to 10 A in 0.025 s, what is the average induced emf in the secondary? What is the change in flux in it if the secondary has 500 turns?

**Data**
- Mutual inductance = \( M = 0.75 \) H
- Change of current in primary coil = \( \Delta I = 10 \) A - 0 A = 10 A
- Time = \( \Delta t = 0.025 \) s
- No. of turns of secondary = \( N_s = 500 \)
- Average induced emf in secondary coil = \( E_s = ? \)
- Change of flux in secondary coil = \( \Delta \phi = ? \)

**Sol.** The emf produced in secondary, when current is changed through primary, is given by:

\[ E_s = M \frac{\Delta I}{\Delta t} \]  

Putting the values, we have:

\[ E_s = 0.75 \times 10 \] \[ \frac{10}{0.025} \] \[ E_s = 300 \text{ V} \] \[ \text{(2)} \]

The emf produced in secondary can also be expressed as:

\[ E_s = N_s \frac{\Delta \phi}{\Delta t} \]

\[ \Delta \phi = \frac{E_s \times \Delta t}{N_s} = \frac{300 \times 0.025}{500} = 0.015 \text{ Wb} \]

P.15.7 A solenoid has 250 turns and its self-inductance is 2.4 mH. What is the flux through each turn when the current is 2 A? What is the induced emf when the current changes at 20 A s\(^{-1}\)?

**Data**
- No. of turns = \( N = 250 \)
- Self-inductance = \( L = 2.4 \) mH = \( 2.4 \times 10^{-3} \) H
- Current = \( I = 2 \) A
- Rate of change of current = \( \frac{\Delta I}{\Delta t} = 20 \) A s\(^{-1}\)
- Flux through each turn = \( \phi = ? \)
- Induced emf = \( E = ? \)

(P.T.6)
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\[ \text{Sol. The magnitude of self induced emf is given by:} \]
\[ E = \frac{L}{\Delta t} \quad (1) \]

Putting values, we get:
\[ E = 2.4 \times 10^{-3} \times 20 \times 2A \]
\[ E = 48.0 \times 10^{-3} V \]
\[ E = 48.0 \text{mV} \]

From Faraday's law, we know that
\[ E = N \frac{\Delta \phi}{\Delta t} \quad (2) \]

Comparing eq. (1) and (2), we have:
\[ N \frac{\Delta \phi}{\Delta t} = \frac{L}{\Delta t} \]
\[ \Delta \phi = \frac{L}{N} \times \Delta I \]

When constant current passes through the coil, then
\[ \phi = \frac{L}{N} \times I \quad (4) \]

Putting the values, we have
\[ \phi = 2.4 \times 10^{-3} \times 20 \times 2A \]
\[ \phi = 1.92 \times 10^{-1} \text{Wb} \]

**P. 15.8:**
A solenoid of length 8.0 cm and cross-sectional area 0.5 cm\(^2\) has 520 turns. Find the self inductance of the solenoid when the core is air. If the current in the solenoid increases through 1.5 A in 0.2 s, find the magnitude of induced emf in it.

**DATA:**
- Length of solenoid \( l = 8.0 \text{ cm} = 0.08 \text{ m} \)
- Cross-sectional area of solenoid \( A = 0.5 \text{ cm}^2 = 0.5 \times 10^{-4} \text{ m}^2 \)
- No. of turns \( N = 520 \)
- No. of turns per unit length \( N = \frac{N}{l} = \frac{520}{0.08} \text{ m}^{-1} \)
- Increase in current \( \Delta I = 1.5 \text{ A} \)
- Time for increase \( \Delta t = 0.2 \text{ s} \)
- Permeability of air \( \mu_0 = 4\pi \times 10^{-7} \text{ Wb A}^{-1} \text{ m}^{-1} \)

Self-inductance for air cored solenoid \( L = \phi \)

**Induced emf:** \[ E = ? \]

**Sol.**

\[ (P.T.S.) \]
Chapter 15: Electromagnetic Induction

The self inductance of air cored solenoid is given by:
\[ L = \mu_0 A \frac{n^2}{\ell} \tag{1} \]

Putting the values, we have:
\[ L = 4\pi \times 10^{-7} \text{ Wb m}^{-1} \times (52 \times 10^{-2})^2 \times 0.08 \times 0.5 \times 10 \text{ m}^{-2} \]
\[ L = 2.12 \times 10^{-4} \text{ H} \tag{2} \]

The self induced emf is given by:
\[ E = L \frac{dI}{dt} \tag{3} \]

Putting the values, we get:
\[ E = 2.12 \times 10^{-4} \text{ H} \times \frac{1.5 \text{ A s}}{0.2} \]
\[ E = 1.6 \times 10^{-3} \text{ V} \]

P.15.9: When current through a coil changes from 100 mA to 200 mA in 0.005 s, an induced emf of 40 mV is produced in the coil. (a) What is the self inductance of the coil? (b) Find the increase in energy stored in the coil.

**Data:**
- Initial current, \( I_1 = 100 \text{ mA} = 0.1 \text{ A} \)
- Final current, \( I_2 = 200 \text{ mA} = 0.2 \text{ A} \)
- Change in current, \( \Delta I = I_2 - I_1 = 0.2 \text{ A} - 0.1 \text{ A} = 0.1 \text{ A} \)
- Time, \( \Delta t = 0.005 \text{ s} \)

Induced emf, \( E = 40 \text{ mV} = 40 \times 10^{-3} \text{ V} \)

(a) Self inductance of the coil, \( L = ? \)

(b) Increase in energy stored in the coil, \( \Delta U_m = ? \)

**Solution:**
(a) The self induced emf in a coil is given by:
\[ E = L \frac{dI}{dt} \tag{1} \]

or:
\[ L = \frac{E}{\frac{dI}{dt}} = \frac{40 \times 10^{-3} \text{ V}}{0.1 \text{ A}/0.005 \text{ s}} = \frac{40 \times 10^{-3} \text{ V}}{0.2} = 2 \times 10^{-2} \text{ H} = 2 \text{ mH} \]

(b) If \( U_m \) is initial and \( (U_m)' \) is the final energy stored in the coil, then change in energy stored is given by:
\[ \Delta U_m = (U_m)' - U_m = \frac{1}{2} L I_1^2 - \frac{1}{2} L I_2^2 \tag{2} \]
\[ \Delta U_m = \frac{1}{2} L (I_2^2 - I_1^2) \tag{2} \]

Putting the values, we get:
\[ \Delta U_m = \frac{1}{2} \times 2 \times 10^{-3} \text{ H} [(0.2)^2 - (0.1)^2] \]
\[ \Delta U_m = 0.03 \times 10^{-3} \text{ J} = 0.03 \text{ mJ} \]
Chapter 15: Electromagnetic Induction

P.15.10: Like any field, the earth’s magnetic field stores energy. Find the magnetic energy stored in a space where the strength of earth’s field is \( 7 \times 10^{-5} \) T, if the space occupies an area of \( 10 \times 10 \, \text{m}^2 \) and has a height of 750 m.

**DATA**
- Strength of magnetic field: \( B = 7 \times 10^{-5} \) T
- Area: \( A = 10 \times 10 \, \text{m}^2 \)
- Height of area above the earth surface: \( h = l = 750 \) m
- Permeability: \( \mu_0 = 4\pi \times 10^{-7} \, \text{Wb/A/m} \)

**Sol.** The magnetic energy density is
\[
U_m = \frac{B^2}{2\mu_0}
\]
\[
= \left( \frac{7 \times 10^{-5}}{2 \times 4\pi \times 10^{-7}} \right)^2 = 1.95 \times 10^{-3} \, \text{J/m}^3
\]
The magnetic energy stored is:
\[
U_m = U_m \times \text{Volume}
\]
\[
U_m = U_m \times \frac{A}{2} = 1.95 \times 10^{-3} \, \text{J/m}^3 \times 10 \times 10 \, \text{m} \times 750 \, \text{m}
\]
\[
U_m = 1.46 \times 10^9 \, \text{J}
\]

P.15.11: A square coil of side 16 cm has 200 turns and rotates in a uniform magnetic field of magnitude 0.05 T. If the peak emf is 12 V, what is the angular velocity of the coil?

**DATA**
- Area of square coil: \( A = 16 \, \text{cm} \times 16 \, \text{cm} = 256 \, \text{cm}^2 = 2.56 \times 10^{-2} \, \text{m}^2 \)
- Number of turns: \( N = 200 \)
- Strength of magnetic field: \( B = 0.05 \) T
- Maximum emf: \( E_0 = 12 \) V
- Angular velocity of the coil: \( \omega = \) ?

**Sol.** The peak value of voltage generated by an AC generator is
\[
E_0 = B \omega N A
\]
or
\[
\omega = \frac{E_0}{BNA} = \frac{12 \, \text{V}}{0.05 \, \text{T} \times 200 \times 2.56 \times 10^{-2} \, \text{m}^2}
\]
\[
\omega = 46.87 \, \text{rad/s}
\]

(Problem T.6)
Chapter 15: Electromagnetic Induction

P.15.12: A generator has a rectangular coil consisting of 360 turns. The coil rotates at 420 rev per min. in a 0.14 T magnetic field. The peak value of emf produced by the generator is 50 V. If the coil is 5.0 cm wide, find the length of the side of the coil.

DATA. No. of turns = N = 360
Angle of frequency = \(\omega = 420 \text{ rev min}^{-1}\)

\[
\omega = \frac{420 \times 2\pi \text{ rad}}{60} = 43.96 \text{ rad s}^{-1}
\]

Strength of magnetic field = \(B = 0.14 \text{ T}\)
Peak value of emf = \(E_0 = 50 \text{ V}\)
Width of coil = \(b = 5.0 \text{ cm} = 0.05 \text{ m}\)
Length of coil = \(l = ?\)

**Sol.** The peak value of emf generated by the generator is:

\[
E_0 = B\omega NA
\]

Where \(A\) is the area of coil given by \(A = lb\)

So,

\[
E_0 = B\omega N lb
\]

or,

\[
l = \frac{E_0}{B\omega NA}
\]

Putting the values, we have

\[
l = \frac{50}{0.14 \times 43.96 \times 0.05 \times 0.05} = 45 \text{ cm}
\]

P.15.13: It is desired to make a generator that can produce an emf of 5KV with 50 Hz frequency. A coil of area \(1 \text{ m}^2\) consisting of 200 turns is used as an armature. What should be the magnitude of the magnetic field in which the coil rotates?

DATA. Electromotive force (emf) = \(E_0 = 5 \text{ KV} = 5000 \text{ V}\)
Frequency = \(f = 50 \text{ Hz}\)
No. of turns = \(N = 200\)
Area of coil = \(A = 1 \text{ m}^2\)
Magnetic field = \(B = ?\)

**Sol.** As,

\[
E_0 = B\omega NA = BNA \times 2\pi f
\]

or,

\[
B = \frac{E_0}{2\pi NA} = \frac{5000}{2 \times 3.14 \times 10 \times 200 \times 1 \text{ m}^2}
\]

\[
B = 0.08 \text{ T}
\]

(P.T.O.)
Chapter 15: Electromagnetic Induction

**P.15.14:** The back emf in a motor is 120 V when the motor is turning at 1680 rev per min. What is the back emf when the motor turns 3360 rev per min?

**DATA:**
- Back emf \( E_1 = 120 \text{ V} \)
- 1st value of angular freq. \( \omega_1 = 1680 \text{ rev min}^{-1} \)
- 2nd value of angular freq. \( \omega_2 = 3360 \text{ rev min}^{-1} \)

**Sol.:**
- As \( E = B \omega NA \) \( \text{(1)} \)
- So \( E_1 = B \omega_1 NA \) \( \text{(2)} \)
- and \( E_2 = B \omega_2 NA \) \( \text{(3)} \)

Dividing eq. (1) by (2), we have:

\[
\frac{E_1}{E_2} = \frac{\omega_1}{\omega_2} \quad \text{(4)}
\]

i.e., the back emf produced in motor is proportional to the angular velocity of coil.

From eq. (4), we have:

\[
E_2 = E_1 \times \frac{\omega_2}{\omega_1} \quad \text{(5)}
\]

Putting the values, we have:

\[
E_2 = 120 \text{ V} \times \frac{3360 \text{ rev min}^{-1}}{1680 \text{ rev min}^{-1}} = 240 \text{ V}
\]

**P.15.15:** A D.C motor operates at 240V and has a resistance of 0.5Ω. When the motor is running at normal speed, the armature current is 15 A. Find the back emf in the armature.

**DATA:**
- Operating voltage \( V = 240 \text{ V} \)
- Resistance \( R = 0.5 \Omega \)
- Armature current \( I = 15 \text{ A} \)
- Back emf \( E = ? \)

**Sol.:**
- The back emf can be calculated as \( V = E + I \times R \)

\[
E = V - I \times R \quad \text{(1)}
\]

Putting the values, we have:

\[
E = 240 - 15 \times 0.5 = 232.5 \text{ V}
\]

(P.T.O.)
Chapter 15: Electromagnetic Induction

**P.15.16:** A copper ring has a radius of 4 cm and resistance of 1.0 mΩ. A magnetic field is applied over the ring, perpendicular to its plane. If the magnetic field increases from 0.2 T to 0.4 T in a time interval of $5 \times 10^{-3}$ s, what is the current in the ring during this interval?

**DATA:**
- Radius of copper ring $r = 4.0$ cm $= 0.04$ m
- Resistance of copper ring $R = 1$ mΩ $= 1 \times 10^{-3}$ Ω
- Increase in magnetic field $\Delta B = B_2 - B_1 = 0.4T - 0.2T = 0.2T$
- Time for increase $\Delta t = 5 \times 10^{-3}$ s
- Current in ring during this interval $I = ?$

**Sol.** The emf induced in the ring is given by Faraday's law:

$E = N \frac{\Delta \Phi}{\Delta t}$  \hspace{1cm} (1)

Here $N = 1$ and $\Delta \Phi = \Delta BA = AB \times \Delta A t \times \mu_0$.

$E = N \times AB \times \mu_0 \frac{\Delta A t}{\Delta t}$ \hspace{1cm} (2)

Putting the values, we have:

$E = 1 \times 0.2 \times 3.14 \times (0.04)^2 \times \frac{5 \times 10^{-3}}{1 \times 10^{-3}}$ \hspace{1cm} (3)

From Ohm's law, the current through the ring is given by:

$I = \frac{E}{R}$

$I = \frac{2.0 \times 10^{-1}}{\frac{5 \times 10^{-3}}{1 \times 10^{-3}}}$ \hspace{1cm} (4)

$= 2.0 \times 10^{-1} \times 1 \times 10^{-3}$ A

$= 2.0 \times 10^{-4}$ A

**P.15.17:** A coil of 10 turns and 35 cm$^2$ area is in a perpendicular magnetic field of 0.5 T. The coil is pulled out of the field in 1.0 s. Find the induced emf in the coil as it is pulled out of the field.

**DATA:**
- No. of turns $N = 10$
- Area $A = 35 \text{ cm}^2 = 35 \times 10^{-4} \text{ m}^2$
- Magnetic field $B = 0.5 \text{ T}$
- Time $\Delta t = 1.0 \text{ s}$
- Induced emf $E = ?$

**Sol.** The magnitude of induced emf by Faraday's law is:

$E = N \frac{\Delta \Phi}{\Delta t}$ \hspace{1cm} (Faraday's Law)

Where $\Delta \Phi = BDA$.

$E = 10 \times 0.5 \times 35 \times 10^{-4} \text{ m}^2 \times \frac{1.75 \times 10^{-2}}{1.0 \text{ s}}$ \hspace{1cm} (P.T.E)
Chapter 15: Electromagnetic Induction

P.15.18: An ideal transformer (step down) is connected to main supply of 240V. It is desired to operate a 12V, 30W lamp. Find the current in the primary and the transformation ratio.

DATA: Primary voltage $V_p = 240\text{V}$
Secondary voltage $V_s = 12\text{V}$
Output power $P_s = 30\text{W}$
Current in primary $I_p = ?$

Transformation ratio $= \frac{N_s}{N_p}$

Sol. As input power = output power, 

$P_p = P_s$

or, $V_p I_p = P_s$

or, $I_p = \frac{P_s}{V_p} = \frac{30\text{W}}{240\text{V}} = 0.125\text{A}$

Transformation ratio is:

$\frac{N_s}{N_p} = \frac{V_s}{V_p}$

$\frac{N_s}{N_p} = \frac{12\text{V}}{240\text{V}} = \frac{1}{20}$