CHP. 7
OSCILLATIONS

● TYPES OF MOTION:
(A) Translatory motion.
Def. "A body moves as a whole from one place to another, all of its particles move in the same direction with the same speed called translatory motion."
In this type of motion, the displacement of a body is known as linear displacement and is measured in metres (SI).
E.g.: All the moving vehicles have translatory motion when they move along a straight line.
(B) Rotatory motion.
Def. "The motion of a body rotating about a fixed line (axis of rotation) is called the rotatory motion."
The displacement of a rotating body is known as angular displacement and is measured in revolutions, degrees, and radians (SI).
E.g.: Motion of a fly wheel.
N.B. A body can simultaneously possess translatory as well as rotatory motion. For example, the wheels of all the moving vehicles have translatory as well as rotatory motion.
(C) Oscillatory or Vibratory motion.
Def. "The motion in which a body moves to and fro about a mean position is called oscillatory or vibratory motion."
The fixed point about which a body performs vibratory motion is called rest or equilibrium or mean position. (P.T. 4)
Vibratory or oscillatory motion is also called simple harmonic motion (SHM) as its wave shape resembles sine or cosine functions which are called harmonic functions. SHM is a particular case of periodic motion.

**Periodic motion.**

Def: "Such a motion which repeats itself after equal intervals of time is called periodic motion."

SHM may be linear or angular.

**(a) Examples of vibratory motion (or SHM).**

**(i) Linear.**

1. The motion of spring-mass system in the horizontal or vertical plane (Fig a, b)
2. A steel ruler clamped at one end to a bench oscillates when the free end is displaced sideways. (Fig c)

![Figure a](image)

![Figure b](image)

![Figure c](image)

3. Inward and outward motion of piston in a cylinder containing a gas when moved inward and released suddenly.
4. Motion of atoms of molecules in solids
5. Plasma oscillations

**(ii) Angular.**

- Motion of simple pendulum about its mean position.
- When a steel ball is rolling in a curved dish, it oscillates about its rest position (Fig e).
3. Motion of a magnet suspended in a uniform magnetic field.

4. Orbital motion

**(b) Production of oscillations.**

In order to get oscillations, a body is pulled away on one side from its rest or equilibrium position and then released. The body begins to oscillate (vibrate) due to restoring force. Under the action of this force, the body accelerates and it overshoots the rest position due to inertia. The restoring force pulls it back. As the restoring force is always directed towards the mean position and so the acc. is also directed towards the rest or mean position.

For example, a violin string produces sound waves in air. So waves are produced due to vibrating bodies i.e; oscillations.

### #7.1 SIMPLE HARMONIC MOTION

**(a) Hooke’s Law.**

Statement: "It states that the displacement $\vec{x}$ produced in an elastic body is directly proportional to the applied force $\vec{F}$ provided its elastic limit is not crossed."

Mathematically,

\[
F \propto x \\
\leftrightarrow F = kx
\]

or 

\[ F = kx \quad \text{(Magnitude)} \quad (1) \]

where $K$ is constant called spring constant for a mass spring system. Its value depends upon the nature of the material of the spring. It is defined as the force per unit extension i.e., $K = \frac{F}{x} \quad (2)$

The SI unit of spring constant is N/m. $K$ is also known as stiffness factor for the spring.
(b) **Elastic restoring force or return force**

Def: "When a force \( F \) is applied to an elastic body to produce a displacement \( \Delta x \) in it, an equal and opposite force acts on it due to elasticity. This force brings the body back to its mean position and is called elastic restoring force."

**Applied force** = \( F = k\Delta x \)  \( \text{(Hooke's law)} \)

**Restoring force** = \( F = -k\Delta x \)  \( \text{(2)} \)

Negative sign is given to this force because it is in the opposite direction to \( \Delta x \).

Applied force is also known as deforming or net or disturbing or accelerating or resultant force.

(c) **Expression for acc. of a body executing SHM under elastic restoring force.**

Consider a mass \( m \) attached to one end of an elastic spring which can move freely on a frictionless horizontal surface as shown in fig(a). When the mass is pulled towards right through a distance \( \Delta x \) from the equilibrium position by a force \( F \), then according to Hooke's law, applied force is directly proportional to displacement \( \Delta x \), i.e.,

\[ F \propto \Delta x \]

or \( F = k\Delta x \)  \( \text{(3)} \)

Due to elasticity the spring opposes the applied force \( F \) which produces the displacement \( \Delta x \). This opposing force \( F_r \) which is equal and opposite to the applied force..."
within elastic limits of the spring:

\[ R = \text{Restoring force} = F = -kx \quad \text{(6)} \]

Let \( a \) is the acc. produced by force \( F \) in mass.

Springs system at any instant, then according to Newton's second law of motion:

\[ F = ma \quad \text{(4)} \]

Comparing eq. (5) and (6), we have

\[ ma = -kx \]

\[ a = -\left(\frac{k}{m}\right)x \quad \text{(7)} \]

\[ a \propto -x \quad \text{(8)} \]

\[ \therefore \frac{k}{m} = \text{const} \]

**SHM.** Def. "Such a type of vibratory motion in which the acc. at any instant of a body is directly proportional to the magnitude of displacement and is directed toward the mean position is called 'Simple harmonic motion' ."

**Examples of SHM.**

1. The vibration of the string of a violin
2. The up and down motion of a loaded elastic string
3. Oscillation of Simple pendulum
4. Motion of a swing

**DIFFERENT TERMS CONNECTED WITH SHM.**

(a) **Instantaneous displacement.**

Def. "The distance moved by the object from its mean position on either side at any instant is known as instantaneous displacement."

It is denoted by \( x \) and is a vector quantity. The **SI unit** is metre \( (m) \)

(b) **Amplitude.**

Def. "The magnitude of the max. displacement of a body on either side from the mean position is called 'amplitude.'"
It is denoted by $x_0$. It is a scalar quantity. The SI unit of amplitude is also metre (m).

(c) Wave form of SHM.

The instant displacement of a body executing SHM is

$$x = r \sin \omega t = r \sin \left( \frac{2\pi}{T} t \right).$$

If we draw a curve between $x$ and $t$, then we get a curve as shown in fig. This curve is a sine curve which is known as wave form of SHM.

Experimental proof.

The experimental arrangement is shown in fig which can be used to record the variation in displacement with time for a mass spring system.

A sheet of paper is placed behind the mass and there is an arrangement to move the paper at a constant speed from right to left. A time scale on the paper is shown by dotted lines. A pen is attached with the vibrating mass which lightly touches the paper. Thus, the pen records (marks) the displacement of the mass on the paper against time. The mass is raised and then released, it performs SHM and the wave form produced on the paper is very similar to sine curve. It is generally as wave form of SHM.

The points $B'$ and $D'$ in the curve correspond to the extreme positions of the vibrating mass and points $A'$ and $C'$.
Displacement may be any distance.

A', C', and E' show its mean position. Thus, the line ACE represents the mean position of the mass on the paper. The amplitude of vibration is thus a measure of the line 'Bb' or 'Dd'.

(d) Vibration.

Def. "One complete round trip of a body about its mean position is called a vibration."

The curve ABCDE in the figure corresponds to the different positions of the mass during one complete vibration. The motion of a vibrating body from its one extreme position back again to the same extreme position is called one vibration. This will correspond to the portion of curve from points B' to E' or from points D' to H'.

(e) Time period.

Def. "The time required to complete one vibration is called time period."

It is denoted by 'T'. The SI unit of time period is second(s).

(f) Frequency.

Def. "The number of vibrations completed by a body in one second is called frequency."

It is denoted by 'f'. The SI unit of frequency is hertz (Hz) or cps or vib/s.

- Relation B/W Time period and frequency.

\[
\text{Time of vibration} = \frac{1}{f} \\
\text{Time of 1 vibration} = \frac{1}{f} \\
\text{But, the time of one vibration} = T \\
T = \frac{1}{f} \text{ or } f = \frac{1}{T}
\]

Time period of a body performing SHM is independent of its amplitude; provided the amplitude is small, such vibrations are called isochronous. SHM is a special case of periodic motion.
Hertz (Hz).

Def. "The frequency of a body is one hertz when it completes one vibration in one second."

Angular frequency:

"The frequency of a periodic circular motion is equal to \(2\pi\) times the no. of cps." It is denoted by \(\omega\).

Mathematically,

\[
\omega = \frac{2\pi}{T}
\]

As \(T = \frac{1}{f}\)

Angular frequency \(\omega\) is basically a characteristic of circular motion. From this we can find the values of instantaneous displacement and instantaneous velocity of a body executing SHM.

### 7.2. SHM. AND UNIFORM CIRCULAR MOTION

**Consider a mass \(m\) attached with the end of a vertically suspended spring vibrates simple harmonically with period \(T\), frequency \(f\), and amplitude \(A_0\). The motion of the mass is displayed by the the pointer \(P\) on the line \(BC\) with \(A\) as mean position and \(B\), \(C\) as (P.T.S).**
extreme position (fig.(a)). Assuming \( A \) as the position of the pointer at \( t=0 \), it will move so that it is at \( B, A', C' \) and back to \( A \) at instants \( T_1, T_2, 3T_4 \) and \( T \) respectively. This will complete one cycle of vibration with amplitude of vibration being \( x_0 = AB = AC \).

Now consider a point \( P \) moving on a circle of radius \( x_0 \) with a uniform angular frequency \( \omega = \frac{2\pi}{T} \). Consider the motion of the point \( N \), the projection of \( P \) on the diameter \( DE \) drawn parallel to the line of vibration of the pointer in fig.(b). Note that the level of points \( D \) and \( E \) is the same as the points \( B \) and \( C \). As \( P \) describes uniform circular motion with a constant angular speed \( \omega \), \( N \) oscillates to and fro on the diameter \( DE \) with the period \( T \). Assuming \( O_1 \) be the position of \( P \) at \( t=0 \), the position of the point \( N \) at the instants \( 0, T_1, T_2, 3T_4 \) and \( T \) will be at the points \( O, D, O, E \) and \( O \) respectively.

A comparison of the motion of \( N \) with that of pointer \( P \) shows that both the motions are identical. Thus the expression of displacement, velocity and acceleration for the motion of \( N \) also holds good for the pointer \( P \), executing SHM.

(a) **Displacement of \( N \)**

It is the distance of the projection \( N \) at any instant from the mean position \( O_0 \).

At this instant, the point \( P \) or radius \( OP \) of fig.(b) makes an angle \( \angle O_1 OP = \theta = \omega t \)

but \( \angle O_1 OP = \angle OPN \) (Alternate angles)

\( \Rightarrow \)
From the right angled triangle $\triangle DONP$, we have
\[
\frac{x}{x_0} = \frac{ON}{OP} = \sin \theta
\]
or
\[
x = x_0 \sin \theta
\]
This will also be displacement of the pointer $P_1$ at the instant $t$.

**Graphical explanation**
The value of $x$ as a function of $\theta$ is shown in Fig. (c). This is the waveform of SHM. The angle $\theta$ gives the state of the system in its vibrational cycle. For example, at the start of the cycle $\theta = 0$. Halfway through the cycle is $180^\circ$ (or $\pi$ radians). When $\theta = 270^\circ$ (or $\frac{3\pi}{2}$ radians), the cycle is three-fourth completed. We call $\theta$ as the phase of the vibration. Thus when quarter of cycle is completed, phase of vibration is $90^\circ$ (or $\frac{\pi}{2}$ radians). Thus phase is also related with circular motion which is an aspect of SHM.

(b) **Instantaneous Velocity**

The linear velocity $v_P$ of the point $P$ in a uniform circular motion at any instant will be directed along the tangent to the circle and its magnitude will be
\[
v_P = r \omega
\]
Here $r = x_0$
\[
\therefore \quad v_P = x_0 \omega \quad (2)
\]
Velocity $v_P$ of the point $P$ on the circle is resolved into two rectangular components.

(P.T.O)
As the motion of 'N' on the diameter 'DE' is due to motion of 'P' on the circle, the velocity of 'N' is actually the component of the velocity 'V_p' in a direction parallel to the diameter 'DE'. This component is 

\[ V_p \sin (90^\circ - \theta) = V_p \cos \theta = \omega_a \omega \cos \theta \]

Thus, the magnitude of the velocity of 'N' or its speed is 

\[ V_N = \omega_a \omega \cos \theta = \omega \omega_a \cos \theta t \quad (3) \]

In \( \triangle ONP \) (right angled triangle),

\[ \cos \theta = \frac{ON}{NP} = \frac{NP}{OP} = \frac{Base}{h/l} \]

\[ NP = OP \cos \theta = OP \cos \omega_a t \quad (4) \]

By Pythagorean theorem

\[ (OP)^2 = (ON)^2 + (NP)^2 \]

or \[ x^2 = z^2 + (NP)^2 \]

\[ OP = \sqrt{x^2 - z^2} \quad (5) \]

Putting the value of \( NP^2 \) in eq. (4), we have

\[ \cos \theta = \frac{z}{\sqrt{x^2 - z^2}} \quad (6) \]

Putting this value in eq. (3), we have

\[ V_N = \omega_a \omega \cos \theta \cdot \omega \omega_a \cos \theta \]

(i) At the mean position, \( z = 0 \), \( V_N = \omega \omega_a \) (max vel.)

(ii) At the extreme position, \( z = \omega_a \), \( V_N = 0 \) (velocity is min)

**Direction of Velocity of 'N'**

The direction of the velocity of 'N' depends upon the value of the phase angle \( \theta \). When \( \theta \) is between \( 0^\circ \) to \( 90^\circ \), the direction is from 'O' to 'D'. When \( \theta \) is between \( 90^\circ \) to \( 270^\circ \), its direction is from 'D' to 'E'. When \( \theta \) is between \( 270^\circ \) to \( 360^\circ \), the direction of motion is from 'E' to 'O'.

As the motion of 'N' on the diameter 'DE' is just similar to the pointer performing SHM, so the pointer 'P' performing SHM is given by eq. (8) in terms of '\( \theta \)'.

(P.T.O.)
**C) Acceleration in terms of \( \omega \)**

When the point \( P \) is moving on a circle, it has an acc.

\[
\mathbf{a}_p = \frac{v_p^2}{x_0}
\]

\[
\mathbf{a}_p = \frac{x_0 \cdot v_p^2}{x_0^2} = x_0 \cdot \omega^2 \tag{8}
\]

This acc. is always directed towards the centre \( O \) of the circle.

At any instant \( t \) its direction will be along \( PO \).

Resolving \( \mathbf{a}_p \) into two rectangular component, we get

- Horizontal component: \( a_p \cos \theta \) (1 to DE)
- Vertical component: \( a_p \sin \theta \) (along \( DE \))

As the point \( N \) moves along the diameter \( DE \) due to motion of \( P \), its acc. will be equal to the component of \( \mathbf{a}_p \) along the diameter \( DE \). Thus, the acc. of \( N \) is given by:

\[
a_N = -a_p \sin \theta = -x_0 \omega^2 \sin \theta \tag{9}
\]

where the negative sign shows that the acc. is directed towards the mean position \( O' \) (i.e., directed from \( N \) to \( O' \))

From fig. 3(e), consider \( \triangle O'NP \),

\[
\sin \theta = \frac{ON}{OP} = \frac{x}{x_0}
\]

\[
x = x_0 \sin \theta \tag{10}
\]

Putting this value in eq. (9), we have:

\[
a_N = -x_0 \omega^2 \frac{x}{x_0} \tag{11}
\]

\[
a_N \propto -x
\]

This shows that the projection \( N' \) of point \( P \) performs SHM. The direction of acc. and displacement \( x \) are always opposite at every instant \( t \). Therefore, the motion of \( N' \) is just a replica of the pointed \( P \) motion.

(i) At mean position, \( x = 0 \) \( \implies a_N = 0 \) (Zero acc.)

(ii) At extreme position, \( x = x_0 \) \( \implies a_N = -x_0 \omega^2 \) (Max. acc.)

(P.T.4)
# 7.3 PHASE:
Def — "The angle $\theta = \omega t$ which specifies the displacement as well as the direction of motion of the point executing SHM or the angle that determines the state of motion of a point in its vibrational cycle is known as phase or phase angle." This angle is obtained when SHM is coupled with circular motion.

Expression for displacement $x$.

The displacement $x = x_0 \sin \theta$ and velocity $v = x_0 \omega \cos \theta$ of projection 'N' of point 'P' executing SHM are determined by the angle $\theta = \omega t$. It is the angle which the rotating radius $OP$ makes with reference direction $OO'$ at any instant 't'. In this special case, we assumed that to start with at $t = 0$, the position of rotating radius $OP$ is along $OO'$, so that the point 'N' is at its mean position and displacement at $t = 0$, is zero.

But in general, we assume that $OP$ initially makes an angle $\phi$ at $t = 0$ with the reference line $OO'$ as shown in fig(3). In time 't', the radius will rotate from $P_{\text{initial}}$ to $P$ by $\omega t$. Now the $OP'$ would make an angle $(\omega t + \phi)$ with $OO'$ at the instant 't' and will have the displacement $ON = x$.

From right angle triangle $OPN$,

$$\frac{ON}{OP} = \sin \angle OPN = \sin \theta$$

but $\angle OPN = \angle (\omega t + \phi)$ (Alternate angles)

$$ON = OP \sin \theta$$

$$x = x_0 \sin (\omega t + \phi) \quad (P.T.\text{-}9)$$
Now phase angle is \( \theta = \omega t + \phi \).

When \( t = 0 \), \( \theta = \phi \).

So \( \phi \) is the initial phase.

If we take initial phase as \( \frac{\pi}{2} \) or \( 90^\circ \), then displacement from eq. (1) can be written as

\[
x = x_0 \sin (\omega t + 90^\circ)
\]

or \( x = x_0 \cos \omega t \) \hspace{1cm} (2)

and \[
\sin (\alpha + \beta) = \cos \beta \hspace{2cm} \text{fig.(b)}
\]

Phase angle is \( \theta = \omega t + \frac{\pi}{2} \).

Eq. (2) also gives the displacement of SHM; but in this case the point 'N' is starting its motion from extreme position instead of the mean position as shown in fig.(b).

### 7.4 A HORIZONTAL MASS SPRING SYSTEM

Consider a mass 'm' attached to one end of the spring whose other end is fixed with a rigid support as shown in fig.

If mass is displaced towards right from its mean position by the application of force and released, thus it execute SHM under restoring force and inertia. Let \( x \) be the displacement of the mass from its mean position at this instant, the restoring force 'F' acting on it is

\[
F = -kx \hspace{2cm} (1)
\]

Also \( F = ma \) \hspace{2cm} (2)

(P.T.S)
Comparing eq. (4) and (2), we have

\[ ma = -Kx \]

\[ a = -\frac{K}{m} \cdot x \quad (3) \]

where \( K \) and \( m \) are constants.

The eq. (3) shows that acceleration is directly proportional to displacement \( x \) and its direction is towards the mean position. Thus the mass \( m \) performs SHM between \( A' \) and \( A'' \) with \( x_0 \) as amplitude. It is similar to the motion of the projection \( N' \) of point \( P \) moving in a circle, so

\[ a = -\omega^2 \cdot x \quad (4) \]

Comparing eq. (3) and (4), we get

\[ -\omega^2 \cdot x = -\frac{K}{m} \cdot x \]

or

\[ \omega^2 = \frac{K}{m} \]

\[ \omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{\text{dimensional constant}}{\text{mass}}} \quad (5) \]

which is the vibrational angular frequency \( \omega \).

with the dimension of

\[ \frac{\left(\omega^2\right)}{\left[\text{dim.}\right]} = \left[\frac{K}{m}\right]^{\frac{1}{2}} = \left[\frac{\text{ML}^{-2}}{\text{LT}^{-2}}\right] = \left[\text{ML}^{-\frac{1}{2}}\right] = \left[\text{T}^{-1}\right] \]

or

\[ [\omega] = \left[\text{T}^{-1}\right] \]

(a) Time period

The time period of the mass executing SHM is given by

\[ \theta = \omega t \]

For one vibration \( \theta = 2\pi \)

\[ 2\pi = \omega t \]

\[ \therefore \quad t = \frac{2\pi}{\omega} \quad (6) \]

Putting the value of \( \omega \) from eq. (5) into eq. (6);

\[ T = \frac{2\pi}{\sqrt{\frac{K}{m}}} = 2\pi \sqrt{\frac{m}{K}} \quad (7) \]

(b) Frequency

Frequency of oscillation is given by

\[ f = \frac{1}{T} \]

\[ f = \frac{1}{2\pi} \sqrt{\frac{K}{m}} \quad (\text{P.T.}) \]
(c) Instantaneous displacement.

The instantaneous displacement of the mass \( m \) is given by:

\[ x = x_0 \sin \omega t \quad \ldots \quad (9) \]

Putting the value of \( \omega \) from eq. (8) into eq. (9), we have

\[ x = x_0 \sin \left( \frac{k}{m} t \right) \quad \ldots \quad (10) \]

(d) Instantaneous velocity.

The instantaneous velocity \( v \) of the point \( N \) performing SHM in case of circular motion is given by:

\[ v = \omega x = \omega \sqrt{x_0^2 - x^2} \quad \ldots \quad (11) \]

Putting the value of \( \omega \) from eq. (8) into eq. (11), we have

\[ v = \sqrt{\frac{k}{m} x_0^2 - x^2} \quad \ldots \quad (12) \]  

or \[ v = \sqrt{\frac{k}{m} x_0 \left( 1 - \frac{x_0^2}{x_0^2} \right)} \quad \ldots \quad (13) \]

or \[ v = x_0 \sqrt{\frac{k}{m} \left( 1 - \frac{x_0^2}{x_0^2} \right)} \quad \ldots \quad (13) \]

(i) At the mean position, \( x = 0 \), \( v_0 = x_0 \sqrt{\frac{k}{m}} \) (max. vel.)

(ii) At the extreme position, \( x = x_0 \), \( v = 0 \) (Zero vel.)

Relation \( b/w \) max. vel. and inst. vel.

Putting the value of \( v_0 = x_0 \sqrt{\frac{k}{m}} \) in eq. (13), we have

\[ v = v_0 \sqrt{\left( 1 - \frac{x_0^2}{x_0^2} \right)} \quad \ldots \quad (14) \]

EXAMPLE 7.1: A block weighing 4 kg extends a spring by 0.16 m from its unstretched position. The block is removed and a 0.5 kg body is hung from the same spring. If the spring is now stretched and then released, what is the period of vibration?

DATA. Mass of the block = \( m_1 = 4 \text{ kg} \)

Length of the stretched spring = \( x = 0.16 \text{ m} \)

Mass of the body = \( m = 0.5 \text{ kg} \)

Period of vibration = \( T = ? \)

\( (P.T.O) \)
**Solution:** By *Hooke's Law*:

\[ F = kx \]

or

\[ k = \frac{F}{x} = \frac{m_1g}{x} \]

\[ k = \frac{4 \times 9.8}{0.12} = 245 \text{ N m}^{-1} \]  \hspace{1cm} (1)

We also know that:

\[ T = 2\pi\sqrt{\frac{m_2}{K}} \]

\[ T = 2 \times 3.14 \times \sqrt{\frac{0.5}{245}} = 0.285 \text{ s} \]

### 7.5 SIMPLE PENDULUM

**Construction:**

“A simple pendulum consists of a small heavy metallic bob suspended from a frictionless support by a light and inextensible string fixed at its upper end in a uniform gravitational field.”

The distance between the point of suspension and the centre of the bob is called the length of the pendulum denoted by \( l \).

When such a pendulum is displaced from its mean position through a small angle \( \theta \) to the position \( B \) and released, it starts oscillating to and fro over the same path.

An ideal pendulum cannot be practically realized.

**Proof for Simple pendulum to execute SHM:**

Let the bob of the simple pendulum be displaced from its mean position \( A \) to the position \( B \) as shown in fig. The forces acting on the bob in this position are:

(i) Weight \( m_2g \) of the bob acting vertically down and

(ii) Tension \( T \) of the string along the direction \( BO \).

\( \text{(P.T.s)} \)
The weight \( mg \) can be resolved into two components, one along the string in the direction \( OB \) and other perpendicular to it along the tangent at \( B \).

- Component of \( mg \) along the string: \( mg \cos \theta \)
- Component of \( mg \) perpendicular to the string: \( mg \sin \theta \)

As there is no motion along the string, \( mg \cos \theta \) balances the tension, i.e.,

\[ T = mg \cos \theta \quad (1) \]

The only force responsible for bringing the bob back to its mean position \( A \) is \( mg \sin \theta \). Therefore, it represents the opposing restoring force \( F \). Thus,

\[ F = -mg \sin \theta \quad (2) \]

Negative sign shows that the motion is towards the mean position.

When the angle \( \theta \) is small and measured in radians,

\[ \sin \theta \approx \theta \quad \text{(rad)} \]

- eq. (2) becomes

\[ F = -mg \theta \quad (3) \]

By Newton's second law,

\[ ma = -mg \theta \quad \therefore F = ma \]

\[ a = -\frac{\theta}{\ell} \quad (4) \]

But \( \theta = \frac{\text{Arc} AB}{\ell} \) \( \therefore s = r \theta \Rightarrow \theta = \frac{s}{r} \)

When \( \theta \) is small, \( s \approx \text{Arc} AB \approx \chi \)

Hence \( \theta = \frac{\chi}{\ell} \quad (5) \)

Putting this value in eq. (4), we have

\[ a = -\frac{\chi}{\ell} \quad (6) \]

\[ \frac{a}{\chi} = -\frac{1}{\ell} \quad : \quad \frac{\chi}{\ell} = \text{const} \]

Eq. (6) shows that acc. of the bob is directly proportional to the displacement \( \chi \) and negative sign shows that it is directed towards the mean position. Thus, the motion of the simple pendulum is **SHM.**

\( (P.T.O.) \)
• **Time period of simple pendulum.**

The acc. of the body, executing SHM. (i.e., projection of a point) is:

\[ a = -\omega^2 x \quad \text{(8)} \]

Comparing eq. (7) and (8), we have

\[ -\omega^2 x = -\frac{\theta}{l} \]

or \[ \omega^2 = \frac{\theta}{l} \quad \text{(9)} \]

The time period of a body executing SHM is given by

\[ T = \frac{2\pi}{\omega} \quad \text{(10)} \]

Putting the value of \( \omega \) from eq. (9) in eq. (10), we have

\[ T = \frac{2\pi}{\sqrt{\frac{\theta}{l}}} = 2\pi \sqrt{\frac{l}{\theta}} \quad \text{(11)} \]

Eq. (11) shows that the time period of simple pendulum depends only on the length of the pendulum and acc. due to gravity. It is independent of the mass of bob attached.

• **Second’s Pendulum.**

Def. "A second pendulum is a pendulum which completes one vibration in two seconds."

As time period of second’s pendulum is 2 s, therefore, its frequency will be:

\[ f = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{\theta}{l}} = \frac{1}{2} \times 0.5 \text{ Hz} \quad \text{(Linear freq.)} \]

and \( \omega = 2\pi f = 2\pi \times 0.5 = \pi \text{ rad/s} \quad \text{(Angular freq.)} \)

It strikes 25 times in a day as there are seconds in it.

• **Use of simple pendulum.**

Simple pendulum provides an accurate method for the determination of \( \theta \) in the laboratory if we neglect buoyancy of air because both \( l \) and \( x \) can be directly measured.

(P.T.O.)
EXAMPLE: What would be the length of a simple pendulum whose period is 1 s at a place where \( g = 9.8 \text{ m/s}^2 \)?

What is the frequency of such a pendulum?

**DATA**

- Time period, \( T = 1 \text{ s} \)
- Acceleration due to gravity, \( g = 9.8 \text{ m/s}^2 \)
- Length of the pendulum, \( l = ? \)
- Frequency of the pendulum, \( f = ? \)

**SOL.**

1. \( T = \frac{2\pi}{\sqrt{g}} \)

   \[ l = \frac{1}{2} \text{(i)} \]

   Or

   \[ l = \frac{4\pi^2}{4 \times (9.8)} = 0.25 \text{ m} \]

2. \( f = \frac{1}{T} = \frac{1}{1.0} = 1 \text{ Hz} \)

**# 7.6 ENERGy CONSERVATION IN SHM**

(a) Max. P.E. Consider the case of a vibrating mass-spring system. When the mass is pulled through some distance, \( x_0 \), against the elastic restoring force, \( F = -kx_0 \).

It is assumed that stretching is done slowly so that

acc. is zero, because change in velocity will be very small. According to Hooke's law:

\[ F = kx_0 \]

- When displacement = 0, force = \( F = 0 \) (At mean)
- When displacement = \( x_0 \), force = \( F = kx_0 \) (At ext)

Thus:

\[ \text{Average force} = F = \frac{F_0 + F_i}{2} = \frac{0 + kx_0}{2} = kx_0 \]

Work done in displacing the mass \( m \) through \( x_0 \) is given by:

\[ W = F \cdot d = \frac{1}{2} kx_0 \cdot x_0 = \frac{1}{2} kx_0^2 \]

This work appears as elastic P.E. of the spring.

Hence

\[ (P.E) = \frac{1}{2} kx_0^2 \]  \( \text{(max. P.E) at the ext. position} \)

(ii) P.E at any instant. At any instant \( t \), if the displacement is \( x \), then P.E at any instant is given by

\[ P.E = \frac{1}{2} kx^2 \]  \( \text{(P.T.)} \)
(iii) Minimum P.E.

P.E is min. if the displacement, x = 0 i.e., when the mass 'm' is at the mean position. Thus
\[ (P.E)_{\text{min}} = \frac{1}{2} kx_0^2 = 0 \]  \hspace{1cm} (3)

(b) (i) K.E. at any instant.

The K.E. of the mass at the instant \( t \) is given by:
\[ K.E = \frac{1}{2} m v^2 \] \hspace{1cm} (4)

As we know, the velocity of the mass attached to the spring at that instant is:
\[ v = \sqrt{\frac{k}{m}} \left( 1 - \frac{x^2}{x_0^2} \right) \] \hspace{1cm} (5)

Putting this value in eq. (4), we have
\[ K.E = \frac{1}{2} m \left[ \frac{k}{m} \left( 1 - \frac{x^2}{x_0^2} \right) \right] \]
or \[ K.E = \frac{1}{2} kx_0^2 \left( 1 - \frac{x^2}{x_0^2} \right) \] \hspace{1cm} (6)

(ii) Max. K.E.

The K.E. of the mass will be max. at the mean position i.e., when \( x = 0 \). Putting in eq. (6), we have
\[ (K.E)_{\text{max}} = \frac{1}{2} kx_0^2 \] \hspace{1cm} (7)

(iii) Min. K.E.

The K.E. is min. when 'm' is at the extreme position 'A' or 'A', when \( x = x_0 \). Putting in eq. (6), we have
\[ (K.E)_{\text{min}} = \frac{1}{2} kx_0^2 \left( 1 - \frac{x_0^2}{x_0^2} \right) = \frac{1}{2} kx_0^2 (0) = 0 \] \hspace{1cm} (8)

(C) Total Energy.

At any displacement \( x \), the energy is partly P.E. and partly K.E. So total energy is given by:
\[ E_{\text{total}} = P.E + K.E \]
\[ = \frac{1}{2} kx_0^2 + \frac{1}{2} kx_0^2 \left( 1 - \frac{x^2}{x_0^2} \right) \]
\[ = \frac{1}{2} kx_0^2 + \frac{1}{2} kx_0^2 \left( 1 - \frac{x^2}{x_0^2} \right) \]
\[ = \frac{1}{2} kx_0^2 \] \hspace{1cm} (9)

(P.T.O.)
(d) Law of conservation of energy in SHM.

The total energy of the vibrating mass and spring remains constant at every instant in its path. This is called as the law of conservation of energy in SHM.

Explanation.

When the mass is at mean position, its K.E is max. but the P.E of the spring is zero. When the mass is at extreme position either A or A', P.E of the spring is max. but the K.E of the mass is zero. In other words, it is wholly P.E at the extreme positions and wholly K.E at the mean position. In any other position, it is partly potential and partly kinetic but the total energy remains the same.

The interchange of energy occurs continuously from one form to the other as the spring is stretched and compressed alternately. The variation of P.E and K.E with the displacement x is essential for maintaining oscillations. The periodic interchange of energy is a basic property of all oscillating systems.

(e) Exchange of P.E and K.E in simple pendulum.

In the case of a simple pendulum, when mass moves from top position to its mean position, its gravitational P.E is converted into K.E. Similarly, this K.E is converted into P.E as the bob rises to the top position. The energy is dissipated due to frictional force such as air friction and consequently the system does not oscillate indefinitely.
Example 7.3. A spring whose spring constant is 80.0 N m\(^{-1}\) vertically supports a mass of 1.0 kg in the rest position. Find the distance by which the mass must be pulled down so that on being released, it may pass the mean position with a velocity of 1.0 m s\(^{-1}\).

**DATA.** Spring constant \(k = 80\) N m\(^{-1}\)

Mass \(m = 1.0\) kg

Velocity of mass \(v = 1.0\) m s\(^{-1}\)

Distance by which mass is pulled down \(x_0 = ?\)

**Sol.** Using the formula:

\[ \omega = \sqrt{\frac{k}{m}} = \frac{80}{1.0} = 8.94 \text{ rad s}^{-1} \]  

Then using the formula:

\[ x_0 = \frac{v}{\omega} = \frac{1.0}{8.94} = 0.11 \text{ m} \]

**7.7 Free and Forced Oscillations (or Vibrations):**

(a) **Free Oscillations.**

Def: "A body is said to be executing free vibrations if it oscillates with its natural frequency without the interference of an external force."

Example. A simple pendulum vibrates freely with its natural frequency that depends only upon the length of the pendulum, when it is slightly displaced from its mean position.

(b) **Forced Oscillations.**

Def: "A body is said to be executing forced vibrations if it is subjected to an external force."

A physical system undergoing forced vibrations is known as a driven harmonic oscillator. (P.T.O)
Examples.

1. If the mass of a vibrating pendulum is struck repeatedly, then forced vibrations are produced.

2. The vibrations of a factory floor caused by the running of heavy machinery is an example of forced vibration.

3. All string instruments produce their loud music due to the forced vibrations of the wooden boards on which they are mounted.

4. When the prongs of a vibrating tuning fork are held over the open end of a resonance tube, forced vibrations are produced in the enclosed air column.

# 7.8 RESONANCE.

Def. — “The marked increase in amplitude of a vibrating body under the periodic force whose period is equal to the natural period is called resonance.”

i.e. Resonance occurs when the frequency of the applied force is equal to one of the natural frequencies of vibration of the forced or driven harmonic oscillator.

The Tacoma Narrows Bridge disaster of 1940 was caused by the bridge being too slender for the wind conditions in the valley. A more tragic accident took place in Angers, France, in 1850, when soldiers marching on a bridge caused sufficient vibration to break the bridge and over 200 of them were killed.

Experiment to demonstrate resonance.

Consider a horizontal rod AB is supported by two strings $S_1$ and $S_2$ as shown in fig. (P.T. 19)
Three pairs of Pendulums $aa', bb'$ and $cc'$ are suspended to this rod. The length of each pair is the same but is different for different pairs.

If one of these pendulums, say $c'$, is displaced in a direction perpendicular to the plane of the paper, a small periodic force acts on all the pendulums through the rod $AB$. The resultant oscillatory motion causes in rod $AB$, a very slight disturbing motion, whose period is the same as that of $c'$. Due to this slight motion of the rod, each of the remaining pendulums ($aa'$, $bb'$ and $cc'$) undergoes a slight periodic motion. This causes the pendulum $c'$, whose length and, hence, period is exactly the same as that of $c'$, to oscillate back and forth with steadily increasing amplitude. However, the amplitude of the other pendulums remain small throughout the subsequent motion of $c'$ and $c''$, because their natural periods are not the same as that of the disturbing force due to rod $AB$.

**Resonant frequency.**

**Def** — The particular freq. which results in the max. amplitude of vibration is called the resonant freq.

**Natural frequency**

**Def** — “When a body is disturbed from its equilibrium position with a specific frequency known as the natural freq. of the body.”

**Natural time period**

**Def** — “When a body vibrates with a natural freq. then its period of vibration is called natural time period.”

($P-T-o$)
Resonance

will also take place if the period of the applied force is any integral multiple of the natural period of the body.

Applications of resonance.

1. Resonance can be used to determine the freq. of a given body. A second body, the natural freq. of which is known, is made to act on the given body.

   If it produces resonance, it is concluded that the given body has the same freq. as the second body.

2. It is used to find natural frequencies of the different bodies.

3. It is used to determine the speed of sound with resonance tube apparatus.

4. Mechanical and electrical system show a good response under phenomenon of resonance.

Examples of resonance.

1. Swing.

   A swing is a good example of mechanical resonance. It is a pendulum with a single natural freq. depending on its length. In the swing, if pushes are given at the correct intervals, which coincide with the period of the swing, the amplitude of the swing can be made quite large. If the pushes are given irregularly, the swing will hardly vibrate.

2. March of soldiers on the bridge.

   If there is a big span of bridge (or a suspension bridge), then the columns of soldiers crossing the bridge are ordered to break their steps. Because if the freq. of their steps coincides with the natural freq. of the bridge, the bridge may be set into vibrations of large amplitude. Thus, the bridge may collapse due to resonance.
3— Tuning of a radio set.

The freq. of a radio set given by \( f = \frac{1}{2\pi \sqrt{LC}} \) can be adjusted by adjusting the values of inductance \( L \) and capacitance \( C \). If we adjust the freq. of our radio set equal to that of the radio station of our interest, we can get the programme of desired station by resonance.

4— Cooking of food by microwave oven.

In microwave oven, the waves produced have a wavelength \( \lambda = \frac{c}{f} \) of 12 cm at a freq. of 2450 MHz. At this freq., the waves are absorbed due to resonance by water and fat molecules in the food, heating them up and so cooking the food evenly and efficiently.

5— Many cities with tall buildings have already refused to allow super sonic planes to fly over them to avoid resonance in buildings.

6— The art of singing and speaking and mechanism of the human ear are excellent natural applications of resonance.

# 7.9 DAMPED OSCILLATIONS:

In dealing with oscillations, we ignored the resistive forces and dissipative effects. In practice, these effects cannot be neglected.

Def— "Such type of oscillation in which the amplitude decreases steadily with time are called \textit{damped oscillations}." or "Such a process in which energy is dissipated from the oscillating system is called \textit{damping} and corresponding oscillations are called \textit{damped oscillations}."
EXPLANATION

In describing the motion of a simple pendulum, frictional effect was completely ignored. As the bob of a simple pendulum moves to and fro, three types of forces come into existence:

(i) Weight of the bob
(ii) Tension in the string
(iii) Viscous drag (i.e., air resistance)

Thus SHM is an idealization as shown in fig (a) which is an undamped harmonic oscillation. In actual the amplitude of the motion of the bob gradually becomes smaller and smaller due to friction and air resistance. fig (b).

Applications of damped oscillations:

1— An application of damped oscillations is the shock absorber of a car which provides a damping force to prevent excessive oscillations. If the shock absorbers are defective then the car will be bouncing even along the smooth roads.

2— Another application of damped oscillations is the shock absorber system of human body. Skier's body moves over the bumpy snow smoothly while his or her thighs and calves act like a damping spring. That is why skiing instructors are reputedly always saying "Bend your knees."
3. A new development in damped oscillations is that racing cars and hydrofoils can now be fitted with active suspensions. These involve computer-aided hydraulic systems in which bumps or waves are sensed, and the suspension system is adjusted accordingly.

4. Computers are used to control the angle of the wave through water to achieve remarkably stable movement of the hovercraft.

# 7.10 SHARPNESS OF RESONANCE

In case of resonance, the amplitude of vibration becomes very large when damping is small. Thus, presence of damping prevents the amplitude from becoming sufficiently large. The amplitude decreases rapidly at a frequency slightly different from resonance freq. Thus, a heavily damped system has a fairly flat curve as shown in an amplitude freq. graph.

In order to observe the damping effect, attach a very light ball such as a pith ball, and another of the same length carrying a heavy ball of equal size as lead bob. They are set into vibrations by a third pendulum of equal length, attached to the same rod. It is observed that the amplitude of the heavy ball (i.e., lead ball) is much larger than that of the pith ball. This shows that the damping effect for the pith ball due to air resistance is much greater than for the lead bob. Thus, the sharpness of the resonance curve of a resonating system depends on energy loss due to friction, i.e., smaller the frictional loss of energy, the sharper the resonance curve.

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SHORT QUESTIONS.

Q7.1 Name two characteristics of S.H.M.

Ans: (i) It is a type of oscillatory motion.
(ii) Acceleration of a vibrating body is directly proportional to the displacement and is always directed towards the mean position i.e. $a = -\alpha x$.
(iii) S.H.M. can be represented by a single harmonic function of sine or cosine in the form of eq.

$$x = A \sin (\omega t + \phi)$$

$$x = A \cos (\omega t + \phi)$$

(iv) The system executing S.H.M. must possess elasticity and inertia.

Q7.2 Does frequency depend on amplitude for harmonic oscillator?

Ans: The frequency of harmonic oscillator is;

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{\mu}} \quad \text{(for mass-spring system)}$$

Both of above eqs. are independent of amplitude but eq.1 depends upon the length of pendulum and acc. due to gravity. Eq.2 depends on its mass. By increasing mass of harmonic oscillator, its frequency decreases.

Q7.3 Can we realize an ideal simple pendulum?

Ans: No, we can not realize an ideal simple pendulum because an ideal simple pendulum consists of a small heavy bob suspended from a rigid, frictionless support by means of a light inextensible string in an air-free atmosphere so that no mechanical energy is dissipated.

Q7.4 What is the total distance travelled by an object moving with S.H.M. in a time equal to its period, if its amplitude is \(A\)?

(P.T.O.)
**Q 7.5** What happens to the period of a simple pendulum if its length is double? What happens if the suspended mass is doubled?

**Ans:** The time period of a simple pendulum is

\[ T = 2\pi \sqrt{\frac{L}{g}} \]  

(i) If length is doubled, eq. (i) becomes

\[ T' = 2\pi \sqrt{\frac{2L}{g}} = \sqrt{2} \cdot 2\pi \sqrt{\frac{L}{g}} = \sqrt{2} \cdot T = 1.41T \]

(ii) Eq. (i) shows that the time period is independent of mass.

**Q 7.6** Does the acc. of a simple harmonic oscillator remain constant during its motion? Is the acc. ever zero? Explain.

**Ans:** The acc. of a simple harmonic oscillator is

\[ a = -\text{const} \cdot x \]

where \( x \) is the displacement from the mean position. As the displacement \( x \) changes during a SHM, its acc. does not remain constant. The acc. becomes zero at the mean position \( (x=0) \) and becomes max. at the extreme positions.

**Q 7.7** What is meant by phase angle? Does it define an angle between max. displacement and the driving force?

**Ans:** The angle which specifies the displacement as well as the direction of motion of the point executing SHM is called phase angle. Thus, it determines the state of motion of the vibrating point.

\[ (P.T.O) \]
It does not define angle between max. displacement and the driving force. It is the angle $\theta = \omega t$, which the rotating radius $OP$ makes with the reference direction $OC$, at any instant as shown in fig.

**Q7.8** Under what conditions does the addition of two SHMs produce a resultant, which is also simple harmonic?

**Ans.** Two SHMs of the same period but of different amplitudes and phases taking place in the same direction can be added to give another SHM. The amplitude of the resultant SHM is equal to the algebraic sum of the amplitudes of the two component SHMs. Its phase angle will also be different.

Given SHMs: $\gamma_1 = a_1 \sin \omega t$, $\gamma_2 = b_1 \sin (\omega t + \phi)$

Resultant SHM is: $\gamma = \gamma_1 + \gamma_2 = a_1 \sin \omega t + b_1 \sin (\omega t + \phi)$

Its converse is also true i.e., a given SHM may be resolved into two components.

**Q7.9** Show that in SHM, the acc. is zero when the velocity is greatest and the velocity is zero when the acc. is greatest.

**Ans.** For a typical SHM,

1. $a = -\omega^2 x$  \(\text{(1)}\)
2. $v = \omega \sqrt{x^2 - x_0^2}$  \(\text{(2)}\)

(i) At mean position, $x = 0$, so $a = -\omega^2 (0) = 0$ and $v = \omega \sqrt{x^2 - x_0^2} = 0 \times x_0$ (max. value)

(ii) At extreme position, $x = x_0$, so $a = -\omega^2 x_0$ (max. value) and $v = \omega \sqrt{x^2 - x_0^2} = \omega (0) = 0$

Hence above statement is true.

\(\text{P.T.O.}\)
Q 7.10 In relation to SHM, explain the equation
\[ Y = A \sin (\omega t + \phi) \]
(ii) \[ \ddot{x} = -\omega^2 x \]

Ans: (i) This equation represents the displacement of an SHM oscillator as a function of time.

Thus, this equation tells that displacement follows a Sine curve, i.e., varies harmonically.

\[ Y \] is instantaneous displacement, \( A \) is the amplitude of the oscillating particle, \( \phi \) is initial phase which tells us about the state of motion, \( (\omega t + \phi) \) is the phase angle made with reference direction and \( \omega t \) is the angle subtended in time \( t \) with angular frequency \( \omega \) starting from initial phase \( \phi \).

(ii) It is the expression of acc. of an object executing SHM. It states that acc. is directly proportional to displacement and it is always directed towards the mean position.

\( \omega \) is the angular frequency of the particle, and \( \ddot{x} \) is the instantaneous displacement from the mean position.

Q 7.11 Explain the relation between total energy, P.E and K.E for a body oscillating with SHM.

Ans: For a body oscillating with SHM,

\[ E_{\text{total}} = P.E + K.E \tag{1} \]

Since total energy of SHM remains constant in the absence of frictional forces, the K.E and P.E are interchanged continuously from one form to another. At mean position, the energy is totally K.E (i.e.; \[ P.E \to 0 \text{ at } x = 0 \])
K.E is max but P.E = 0. At the extreme positions the K.E is completely changed into P.E. In between, it is partly P.E and partly K.E.

Q7.12 Describe some common phenomena in which resonance plays an important role.

Ans: Following are the common phenomena in which resonance plays an important role.

(i) Tuning of a radio. By tuning a dial, the natural freq. of an A.C in the receiving circuit is made equal to the freq. of the wave broadcast by the desired station. When the two freq. match, resonance occurs and we hear the programme of desired station.

(ii) Swing. If the swing is pushed after regular intervals of time (equal to the period of swing), its motion will increase with every push. If the pushes occur at irregular intervals, the swing will hardly vibrate.

(iii) Microwave oven. The wave produced in this type of oven have a wavelength of 12 cm at a freq. of 2450 MHz. At this freq. the waves are absorbed due to resonance by water and fat molecules in the food resulting in efficient and evenly heating and cooking of the food.

(iv) Musical strings. In the musical strings when the freq. of enclosed air column in the wooden boxes under the strings becomes equal to the string freq., due to resonance a loud sound of music is heard.

Q7.13 If a mass-spring system is hung vertically and set into oscillations, why does the motion eventually stop?

Ans: A SHM eventually stops due to friction, air resistance and some other damping forces. Thus, mechanical energy of the system is wasted into heat and consequently the system does not oscillate indefinitely and eventually stops.
NUMERICAL PROBLEMS

P.7.1 A 100 g body hung on a spring elongates the spring by 4 cm. When a certain object is hung on the spring and set vibrating its period is 0.568 s. What is the mass of the object pulling the spring?

DATA:
- Mass of the body = \( m = 100 \text{ g} = 0.1 \text{ kg} \)
- Extension in the spring = \( x = 4 \text{ cm} = 0.04 \text{ m} \)
- Time period = \( T = 0.568 \text{ s} \)

Mass of the object = \( m' = ? \)

Sol. According to Hooke's law,
\[
F = kx \quad \text{or} \quad k = \frac{F}{x} \quad (1)
\]
\[
k = \frac{mg}{x} = \frac{0.1 \times 9.8}{0.04} = 24.5 \text{ N m}^{-1}
\]
Now for mass-spring system, we know that
\[
T = 2\pi \sqrt{\frac{m}{k}}
\]
or
\[
T^2 = 4\pi^2 \frac{m}{k}
\]
\[
\Rightarrow \quad m' = \frac{T^2 k}{4\pi^2}
\]
\[
m' = (0.568)^2 \times 24.5 = 0.2 \text{ kg} \quad \text{or} \quad 200 \text{ g}
\]

P.7.2 A load of 15 g elongates a spring by 2 cm. If body of mass 294 g is attached to the spring and is set into vibration with an amplitude of 10 cm, what will be its (i) period (ii) spring constant (iii) max. speed of its vibration?

DATA:
- Load = \( m = 15 \text{ g} = 0.015 \text{ kg} \)
- Extension in the spring = \( x = 2 \text{ cm} = 0.02 \text{ m} \)
- Mass attached to the spring = \( m' = 294 \text{ g} = 0.294 \text{ kg} \)
- Amplitude = \( x_0 = 10 \text{ cm} = 0.1 \text{ m} \)

(i) Time period = \( T = ? \)
(ii) Spring constant = \( k = ? \)
(iii) Max. Speed = \( v_0 = ? \)

(P.T.5)
Sol. (i) According to Hook’s law,
\[ F = kx \]  \hspace{1cm} \text{(1)}

But, \[ F = W = mg \]  \hspace{1cm} \text{(2)}

We have \[ kx = mg \]

So, \[ k = \frac{mg}{x} = \frac{0.015 \times 9.8}{0.02} = 7.35 \text{ N m}^{-1} \]

(ii) Time period of the mass spring system is
\[ T = 2\pi \sqrt{\frac{m}{k}} \]  \hspace{1cm} \text{(3)}

\[ T = 2 \times 3.14 \times \sqrt{\frac{0.294}{7.35}} = 1.7 \text{ s} \]

(iii) We know that
\[ v = \sqrt{\frac{k}{m}} \]  \hspace{1cm} \text{(4)}

In this case \( m = \text{load} + \text{mass} = 0.015 + 0.29 = 0.309 \text{ kg} \)  \hspace{1cm} \text{(5)}

Putting the values, we get
\[ v = 0.1 \sqrt{\frac{7.35}{0.309}} = 0.488 \text{ m s}^{-1} = 48.8 \text{ cm s}^{-1} \]

P.7.3 An 8 kg body executes SHM with amplitude 30 cm. The restoring force is 60 N when the displacement is 30 cm. Find (i) Period (ii) Acceleration, speed, K.E and P.E when the displacement is 12 cm.

DATA. Mass of body = \( m = 8.0 \text{ kg} \)
Amplitude = \( x_0 = 30 \text{ cm} = 0.3 \text{ m} \)
Restoring force = \( F = 60 \text{ N} \)
Displacement = \( x = 30 \text{ cm} = 0.3 \text{ m} \)

(i) Period = \( T = ? \)
(ii) Acceleration = \( a = ? \)
(iii) Speed = \( v = ? \)
(iv) K.E = ?
(v) P.E = ? (When the displacement, \( x = 12 \text{ cm} \))

Sol. (i) According to Hook’s law,
\[ F = kx \]

\[ k = \frac{F}{x} = \frac{60}{0.3} = 200 \text{ N m}^{-1} \]  \hspace{1cm} \text{(6)}

\[ (P.T.6) \]
Using the formula for the time period of a spring mass system,
\[ T = 2\pi \sqrt{\frac{m}{k}} \]
\[ T = 2 \times 3.14 \times \sqrt{\frac{8}{2 \times 0.3}} = 1.3 \text{ s} \]

(ii) As we know
\[ F = -kx \]  
and \[ F = ma \]

or \[ ma = -kx \]

\[ \ddot{x} = \frac{-kx}{m} \]  
\[ x = 12 \text{ cm} = 0.12 \text{ m} \]
\[ a = \frac{-200 \times 0.12}{0.12} = -3 \text{ m/s}^2 \]

Negative sign indicates that acc. is directed towards the mean point.

(iii) The speed of any body executing SHM is given by
\[ v = \omega \sqrt{x_0^2 - x^2} \]

But \[ \omega = \frac{2\pi}{T} = \frac{2 \times 3.14}{1.3} = 4.82 \text{ Hz} \]
\[ v = 4.82 \sqrt{(0.3)^2 - (0.12)^2} \]
\[ v = 1.33 \text{ m/s} \]

(iv) For K.E, we know that
\[ K.E = \frac{1}{2} kx^2 \left(1 - \frac{x^2}{x_0^2}\right) \]
\[ = \frac{1}{2} \times 200 \times 0.3 \left(1 - \frac{(0.12)^2}{(0.3)^2}\right) \]
\[ K.E = 7.56 \text{ J} = 7.6 \text{ J} \]

(v) For P.E, we know that
\[ P.E = \frac{1}{2} kx^2 \]
\[ = \frac{1}{2} \times 200 \times (0.12)^2 \]
\[ = 1.44 \text{ J} \]

(P.T.O.)
P.7.4 A block of mass 4.8 kg is dropped from a height of 0.8 m on to a spring of spring constant \( k = 1960 \, \text{N/m} \). Find the max. distance through which the spring will be compressed.

**DATA**: mass of the block = \( m = 4 \, \text{kg} \)

\[ h = 0.8 \, \text{m} \]

\[ k = 1960 \, \text{N/m} \]

**Sol.** We know that

\[ P.E = mgh \tag{1} \]

\[ P.E = 4 \times 9.8 \times 0.8 = 31.36 \, \text{J} \tag{2} \]

When the block is dropped on the spring, the spring will be compressed through max. distance \( x_0 \) due to the P.E. of the block. Thus,

\[ P.E = \frac{1}{2} k x_0^2 \tag{3} \]

\[ x_0^2 = \frac{2 \times P.E.}{k} \]

\[ x_0^2 = \frac{2 \times 31.36}{1960} = 0.018 \, \text{m}^2 \]

\[ x_0 = \sqrt{0.018} = 0.42 \, \text{m} \]

P.7.5 A simple pendulum is 50 cm long. What will be its freq. of vibration at a place where \( g = 9.8 \, \text{m/s}^2 \)?

**DATA**: Length of simple pendulum = \( l = 50 \, \text{cm} = 0.5 \, \text{m} \)

Acc. due to gravity = \( g = 9.8 \, \text{m/s}^2 \)

Freq. of simple pendulum = \( f = ? \)

**Sol.** Time period of simple pendulum is,

\[ T = 2\pi \sqrt{\frac{l}{g}} \]

\[ T = 2 \times 3.14 \sqrt{\frac{0.5}{9.8}} = 1.41 \, \text{s} \tag{1} \]

We know that

\[ f = \frac{1}{T} \]

\[ f = \frac{1}{1.41} = 0.71 \, \text{Hz} \] (P.T.S)
P.7.6 A block of mass 1.6 kg is attached to a spring with spring constant 1000 N m\(^{-1}\) as shown. The spring is compressed through a distance of 2 cm and the block is released from rest. Calculate the velocity of the block as it passes through the equilibrium position, \(x=0\) if the surface is frictionless.

**DATA**
- Mass of block: \(m=1.6\) kg
- Spring constant: \(K=1000\) N m\(^{-1}\)
- Max. displacement: \(x_o=2\) cm = 0.02 m
- Vel. at mean position: \(V=\) ?

**Sol.** As the velocity is max. at the mean position, so the formula is given by:

\[
V = x_o \sqrt{\frac{K}{m}}
\]

Putting the values,

\[
V = 0.02 \times \sqrt{\frac{1000}{1.6}} = 0.5 \text{ m s}^{-1}
\]

P.7.7 A car of mass 1300 kg is constructed using a frame supported by four springs. Each spring has a spring constant 20000 N m\(^{-1}\). If two people riding in the car have a combined mass of 160 kg, find the freq. of vibration of the car, when it is driven over a pot hole in the road. Assume the weight is evenly distributed.

**DATA**
- Mass of car: \(m_1=1300\) kg
- Mass of two people: \(m_2=160\) kg
- Spring constant of each spring: \(K=20000\) N m\(^{-1}\)
- Frequency of vibration: \(f=\) ?

**Sol.** The formula for the time period of mass spring system is:

\[
T = 2\pi \sqrt{\frac{m}{K}}
\]

But \(f = \frac{1}{T}\)

\[
f = \frac{1}{2\pi} \sqrt{\frac{K}{m}}
\]

\[
f = \frac{1}{2\times3.14} \sqrt{\frac{80000}{1460}} = 1.18 \text{ Hz}
\]

Hence \(f = 1.18 \text{ Hz}\)

\((P.T.O.)\)
P. 7.8 Find the amplitude, freq. and period of an object vibrating at the end of a spring, if the eq. for its position as a function of time is \( x = 0.25 \cos \left( \frac{\pi}{8} t \right) \). What is the displacement of the object after 2.0 s?

**DATA.** Time = \( t = 2.0 \) s

(i) Amplitude = \( a_0 = ? \)  
(ii) Frequency = \( f = ? \)  
(iii) Period = \( T = ? \)  
(iv) Displacement = \( x = ? \)

**Sol.** (i) The general eq. of SHM is

\[ x = a_0 \cos \omega t \tag{1} \]

Comparing the given eq.

\[ x = 0.25 \cos \left( \frac{\pi}{8} t \right) \tag{2} \]

Amplitude = \( a_0 = 0.25 \) m

(ii)

\[ \omega = \frac{\pi}{8} \]

But \( \omega = 2\pi f \)

\[ f = \frac{\omega}{2\pi} = \frac{\pi/8}{2\pi} = \frac{1}{16} \text{ Hz} \]

(iii) The time period is given as

\[ T = \frac{1}{f} = \frac{1}{1/16} = 16 \text{ s} \]

(iv) Displacement after 2.0 seconds:

\[ x = 0.25 \cos \left( \frac{\pi}{8} \times 2 \right) \]

\[ x = 0.25 \cos \left( \frac{\pi}{4} \right) \]